NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

DISTRIBUTIONAL ANALYSIS
OF INVENTORY
DEMAND OVER LEADTIME

by

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June 1982

Thesis Advisor: Charles F. Taylor, Jr.

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Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Distributional Analysis of Inventory Demand Over Leadtime		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis June 1982
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Mark Lee Yount		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE June 1982
		13. NUMBER OF PAGES 125
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Inventory demand distributions, Mixed Bernoulli distributions, Bootstrap, Data analysis, Resampling, Probabilistic models, Statistical models

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

#20 - ABSTRACT - (Continued)

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Distributional Analysis of Inventory Demand Over Leadtime

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MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
June, 1982

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

I. INTRODUCTION

Inventory system performance[1] in a situation with random demand depends upon reorder point[2] computations. Reorder point computations depend upon the probabilistic model chosen to represent the inventory system in that the reorder point is composed of two parts: 1) The expected demand during a procurement leadtime and 2) The safety level[3] determined from the probabilistic model of demand over leadtime. Probabilistic models are generally only utilized to represent demand over procurement leadtime since that is the only time period when stockouts[4] potentially occur. The chosen distribution is then a conditional distribution given procurement leadtime. The Navy has historically used three distributions of demand for models utilized at the Inventory Control Point (ICP) echelon of the Navy Supply System. The current probabilistic models and the average annual demand that is used to determine which model to utilize are displayed in Table 1.[5]

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[1] See Appendix A for definition of "System Performance".

[2] See Appendix A for definition of "Reorder Point".

[3] See Appendix A for definition of "Safety Level".

[4] See Appendix A for definition of "Stockouts".

[5] The Aviation Supply Office (ASO) has recently changed to the use of the normal distribution as the model of choice for

Table 1: Current Inventory Control Point
Probabilistic Demand Over Leadtime Models

Distribution	Average Annual Demand Range	
Poisson	0 - 1	Low
Negative Binomial	1 - 20	Medium
Normal	> 20	High

If the actual quarterly demand records for an ICP are examined, one striking characteristic that is present in nearly all records is the high incidence of zero observations. This is not surprising when one considers the echeloning of the supply system and the fact that users only place a demand on the wholesale system as their reorder points are reached. Since most activities stock at least one quarter's expected demand for an item, reorders are expected at most four times a year. By the natural phase differences of each activity's reorder actions, the ICP may experience zero demands during a given quarter for even high-demand items, and will surely often experience quarters of zero demands for medium- and low-demand items. A previous study of Pacific Fleet Combat Stores Ships' demand data [Ref. 1] found that the frequency of zero observations in those demand

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all demand categories. Ships Parts Control Center (SPCC) still uses all the distributions in Table 1.

records appeared to have a low correlation with the overall demand level (i.e. high, medium or low).[6]

A desirable model for an inventory system should compute reorder levels that accurately correspond to a specified stockout risk based on the level of actual or anticipated demand for each item. If such a model were available, the inventory system could avoid the added expense of either having too much or too little stock. One problem with the distributions in Table 1 is that none can account for the probability mass at zero; therefore, none can accurately compute reorder levels.

This study explores the use of compound probability models and their potential to more accurately achieve the goals presented above by explicitly accounting for the probability mass at zero. The results obtained should be considered as a first step in the exploration of a class of models which have heretofore received little attention in the context of modeling inventory demand.

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[6] See Section II.B.1 for a more detailed discussion of this study.

II. BACKGROUND

A. CURRENT MODEL SELECTION TECHNIQUES

In the past, the usual approach of choosing the probabilistic model for lead time demand was based on matching the empirical cumulative distribution function determined from the actual demand to various theoretical cumulative distribution functions. The shapes of the two curves are compared using the Kolmogorov-Smirnov Statistical Goodness of Fit test.[7] The Chi-Squared test has historically not been used because it is not very powerful when the number of observations is small (supply demand data is retained for at most twelve quarters and usually only eight quarters at the ICP level). The Kolmogorov-Smirnov test provides only a relative measure of goodness of fit, but it is usable with very small sample sizes. The goal then is to select the model with the best agreement between the empirical and theoretical cumulative distribution function curves.

The Navy Fleet Material Support Office (FMSO) has utilized the above Kolmogorov-Smirnov test to evaluate several theoretical distributions. Most recently [Ref. 2] FMSO

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[7] A good description of the Kolmogorov-Smirnov one and two-sample tests may be found in Siegel, S., Non Parametric Statistics, p.47-52 and 127-136, McGraw Hill, 1956.

evaluated seven distributions[8] at the 90% confidence level using demand data from NAS Brunswick. The use of demand data from a stock point vice an ICP assumes the underlying demand behavior at the stock point and ICP are similar. Though this was never validated directly, the NAS Brunswick demand data were deemed to have "unique qualities better suited for demand analysis" as follows:

1. Twelve quarters of demand and demand frequency data were available per item, 50% more than available in ICP files,
2. New item identifiers that permit the selection of only steady state demand items were available,
3. Reliability of the data was established from prior FMSO studies.

The FMSO analysis showed that Navy demand patterns are very poorly modeled by any of the seven distributions studied. No new models were proposed and no attributes of the demand patterns were indicated as the most significant factor in the failure of the standard distributions to model Navy demand patterns.

There are several problems with the way the Kolmogorov-Smirnov test was used in the FMSO study:

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[8]Distributions tested were: Normal, Negative-Binomial, Poisson, Logistic, LaPlace, Gamma and Uniform.

1. The procedure is not strictly applicable in the case of discrete distributions such as the Poisson and negative binomial.
2. The test assumes that the parameters of the distribution being tested are completely specified in advance, i.e. not estimated from the data. (Lilliefors [Ref. 3] developed alternate tables for use in testing the exponential distribution when parameters are estimated from the data. This procedure was used in Reference 1.)
3. The Kolmogorov-Smirnov test evaluates goodness-of-fit over the entire range of the distribution. For inventory problems, the region of greatest interest is the right-hand tail of the distribution.

B. PREVIOUS RESEARCH

Research into distributions which have properties that more closely model actual demand patterns has been very limited and, when conducted, often did not explore anything but the usual standard distributions. Two projects though stand out because of their innovative use of non-standard distributions and the good fit achieved.

1. Bernoulli-exponential Distribution

In Reference 1, Pacific Fleet Combat Stores Ship demand was modeled using the mixed Bernoulli-exponential

distribution. This distribution was chosen because it was observed that nearly all line items had an unusually high occurence of demands of zero. Analysis of the data indicated that with high probability, the number of non-zero observations for a given item was unrelated to the average value of those non-zero observations. The conclusion was that the demand process could be thought of as two independent subprocesses with one process determining whether a demand would occur or not and the other determining the quantity of the demand, given the demand did occur. The former process was modeled as a Bernoulli process with parameter p, p being the probability that a demand did occur. The latter process was modeled as a continuous exponential process based on exploratory data analysis. The resulting probability distribution function for the number of units demanded in a leadtime for the Bernoulli-exponential distribution is given by:

$$h(x) = \begin{cases} 1 - p & , x = 0 \\ p\lambda \exp(-\lambda x), & x > 0, \end{cases} \tag{1}$$

where:

$1/\lambda$ = the expected value of demand, given that the demand is greater than zero.

p = probability that demand does occur.

The complementary cumulative distribution function is given by:

$$H(x) = p \exp(-\lambda x), \quad x \geq 0, \tag{2}$$

where H(x) is the probability that demand will not exceed x; this is equivalent to the complementary cumulative distribution function used by Hadley and Whitin [Ref. 4].

The hypothesis that the demand of Combat Stores Ships could be modeled according to the above probability distribution was tested using the Kolmogorov-Smirnov test described earlier plus tests on the theoretical risk and the observed risk for values of risk in the upper right hand tail area of the distribution. The results of these tests on five independent samples from the demand data available provided strong evidence that the Bernoulli-exponential distribution describes the demand for any given stock item very well.

2. <u>Logarithmic-Poisson-Gamma Distribution</u>

A model for the distribution of demand during procurement leadtime [Ref. 5] was derived under the following assumptions:

a. Requisitions occur according to a stationary Poisson Process,

b. Requisition sizes follow a logarithmic distribution,

c. Leadtime is a random variable with the gamma distribution.

The resulting probability function for the number of units demanded in a leadtime is:

$$h(x) = \left(\frac{\beta}{\lambda + \beta}\right)^{\alpha} \frac{\theta}{x!} \sum_{k=1}^{x} \left(\frac{c}{\lambda + \beta}\right)^{k} S_{x,k} \sum_{j=1}^{k} S_{k,j} \tag{3}$$

for $x = 1, 2, 3, \ldots$

and

$$h(0) = \left(\frac{\beta}{\lambda + \beta}\right)^{\alpha}$$

where:

α, β = parameters of the Gamma distribution defining leadtime,

θ = parameter of the logarithmic distribution solved for by interval bisection from $E(X) = -\theta / (1 - \theta)\ln(1 - \theta)$,

λ = requisition arrival rate,

$c = -\lambda / \ln(1 - \theta)$

$S_{x,k}$ = Stirling numbers of the first kind computed from the recursion

$S_{x,k} = S_{x-1,k-1} + (x - 1) S_{x-1,k}$

for $k = 1, 2, \ldots, x$ and $x = 1, 2, \ldots$

with $S_{x,0} = 0$ for all x

This distribution is called the Logarithmic-Poisson-Gamma (LPG). It is defined from four parameters α, β, Θ, and λ (Θ and λ determine c).

For more efficient computations, a recursion equation for integer α was developed.

$$h(x) = \sum_{k=1}^{x} T_{x,k} \tag{4}$$

where:

$$T_{x,k} = \frac{\Theta}{x} \left[C_2(\alpha + k - 1) T_{x-1,k-1} + (x - 1) T_{x-1,k} \right]$$

$$C_2 = c \,/\, (\lambda + \beta)$$

and Θ, c, α, β and λ defined as in (3).

Computing reorder points from fractiles of the LPG distribution, even using the simplified version of (4), is generally too time consuming for many real applications. As a result, an approximation was developed and tested that used a scaled version of the Poisson distribution to approximate the negative binomial distribution. The resulting approximation is:

$$p\{Z(t) = kx\} \approx \frac{(\alpha + x - 1)!}{x!\,(\alpha - 1)!} \left(\frac{\beta}{\mu + \beta}\right)^{\alpha} \left(\frac{\mu}{\mu + \beta}\right)^{x} \tag{5}$$

for x = 0, 1, 2, ...

where:

Z(t) = number of units demanded in time t

$$\mu = c\Theta$$

α, β, c and Θ defined as in (3).

The authors conducted subjective tests of goodness of fit to their LPG model and its derivatives using Air Force consumable items and concluded, using as an example one item, that there was a "very close agreement between the observed and the predicted cumulative distribution functions for this item."

III. PROPOSED MODELS

A. COMPOUND BERNOULLI MODELS IN GENERAL

The results of the previous research indicate that a potential exists for improving inventory performance by the use of slightly more complex models. The focus of this study is on the compound Bernoulli models, leaving further analysis of the LPG model and its derivatives to others. The use of compound Bernoulli models has several advantages over standard simple distributions:

1. The model, by necessity, has an added parameter whose sole function is to estimate the probability of a non-zero demand.
2. For simple models defined only over the positive real numbers, the addition of the Bernoulli parameter simply results in the scaling of the cumulative distribution, thus adding little additional complexity to the model.
3. By explicitly accounting for zero demands, compound models account for more of the observed variance than do simple models.

Compound models are not without disadvantages:

1. Each parameter added to the model must be estimated for each individual stock item. For a large inventory

system such as the Navy's, this equates to considerable additional computational requirements.

2. Additional parameters must be stored for future retrieval or else computed from stored data when required, thus either extending computational time or else requiring additional online storage space.

The Bernoulli compound models all have similarly shaped cumulative distribution functions. Figure 1 is a sample distribution function for the compound Bernoulli-lognormal distribution with a Bernoulli factor of p equal to 0.6, a mean of 1.0 and a standard deviation of 0.5. Note that the distribution has a mass of zero for negative observations, a mass of 1-p at zero and the usual cumulative distribution function shape for positive observations but of mass p vice mass one.

B. MODEL DESCRIPTIONS

Three compound Bernoulli models were formulated for evaluation. Each model was derived from the base distribution after compounding with the Bernoulli process. An additional requirement was made that the inverse of the compound model must be computationally relatively simple.

1. Bernoulli-exponential Distribution

This distribution was derived in Reference 1 and the derivation will not be repeated here. Equations (1) and (2)

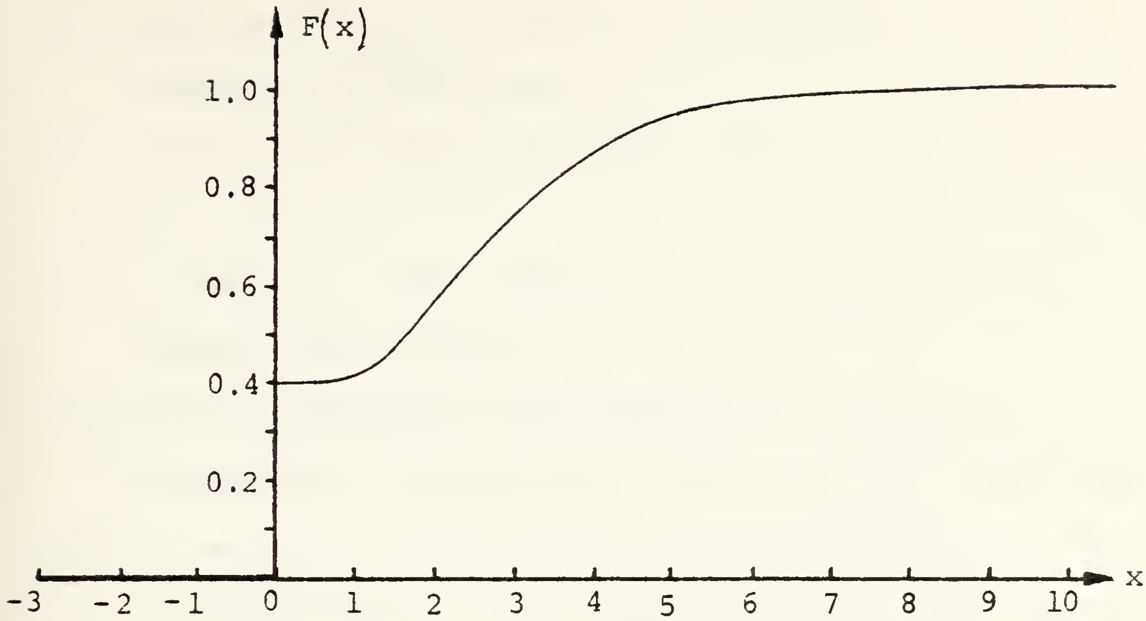

Figure 1: Compound Cumulative Distribution Function, Bernoulli-lognormal Distribution

are the probability density and complementary cumulative distribution functions respectively. The Bernoulli-exponential inverse complementary cumulative distribution function is computed as:

$$x = \begin{cases} 0 & , H(x) \geq p \\ \mu[\ln(p) - \ln(H(x))], & H(x) < p \end{cases} \tag{6}$$

The parameters of this distribution are estimated from the sample data as follows:

$\hat{p}$ = number of non-zero demand observations in sample divided by sample size,

$\hat{\mu} = \bar{x} / \hat{p}$, where $\bar{x}$ is the entire sample mean,

or

$\hat{\mu} = \bar{x}$, where $\bar{x}$ is the sample mean of only the non-zero demand observations.

2. Bernoulli-lognormal Distribution

The lognormal distribution was chosen as a candidate for testing because of several desirable properties:

a. It is defined for all real numbers greater than zero,

b. Its inverse is readily computed using inverse normal approximations that are well documented for accuracy,

c. Alone, the lognormal distribution does not allow the case of zero demand, but when combined with the Bernoulli distribution, the compound Bernoulli-lognormal distribution defines all the expected demand values greater than or equal to zero.

Utilizing the Bernoulli parameter p as a scaling factor for the lognormal density function, the compound Bernoulli-lognormal density function may be expressed as:

$$h(x) = \begin{cases} 1 - p & , x = 0 \\ p\phi\left(\dfrac{\ln(x) - \mu}{\sigma}\right), & x > 0 \end{cases} \tag{7}$$

where:

p = probability of a non-zero demand,

μ = mean of the natural log of all demands greater than zero,

σ = Standard deviation of the natural log of all demands greater than zero,

ϕ = Standard normal density function.

Integrating (7) results in the complementary cumulative distribution function given by:

$$H(x) = \begin{cases} p & , x = 0 \\ p\left[1 - \Phi\left(\dfrac{\ln(x) - \mu}{\sigma}\right)\right], & x > 0 \end{cases} \qquad (8)$$

where:

Φ = Standard normal cumulative distribution function.

The Bernoulli-lognormal inverse cumulative distribution function is easily derived from (8) as:

$$x = \begin{cases} 0 & , H(x) \geq p \\ \exp[\mu + \sigma\Phi^{-1}(1 - H(x)/p)], & H(x) < p \end{cases} \qquad (9)$$

The parameters are estimated from the sample data in the usual manner:

$\hat{p}$ = number of non-zero demand observations in sample divided by sample size,

$\hat{\mu} = \bar{x}$, where $\bar{x}$ is the sample mean of the natural log of all non-zero demand observations,

$\hat{\sigma} = s$, where s is the sample standard deviation of the natural log of all non-zero demand observations. An alternate method for estimating the mean and standard deviation of the lognormal distribution is to use the method of moments. The procedure in effect, transforms the mean and standard deviation obtained from the untransformed sample data vice transforming the data first and then computing the sample mean and standard deviation. This method involves fewer logarithms and will run faster on a computer. The procedure is described in Appendix D.

3. Bernoulli-logistic Distribution

The logistic distribution is a pseudo-normal distribution that is similar in shape to the normal, but is easier to handle mathematically. Its density function is defined as [Ref. 6]:

$$f(x) = \frac{\pi}{4\sqrt{3}\sigma} \operatorname{sech}^2\left(\frac{\pi(x-\mu)}{2\sqrt{3}\sigma}\right), \quad -\infty < x < \infty, \tag{10}$$

while its cumulative distribution function is defined as:

$$F(x) = \frac{1}{2}\left[1 + \tanh\left(\frac{\pi(x-\mu)}{2\sqrt{3}\sigma}\right)\right], \quad -\infty < x < \infty, \tag{11}$$

where:

μ = mean of the entire sample,

σ = standard deviation of the entire sample.

With the logistic distribution, it is no longer a simple matter of scaling the distribution by the Bernoulli p parameter, since this distribution is defined over the entire real line and not just on the positive half. Consequently, the distribution requires a new constant of integration to replace the 1/2 used in (11). Integrating (10) from 0 to ∞ provides the new constant as:

$$\int_0^\infty \operatorname{sech}^2\left(\frac{\pi(x-\mu)}{2\sqrt{3}\,\sigma}\right)dx = 1 + \tanh\left(\frac{\pi\mu}{2\sqrt{3}\,\sigma}\right) \tag{12}$$

which, when compounded with the Bernoulli distribution, gives the Bernoulli-logistic distribution density function as:

$$h(x) = \begin{cases} 1 - p & , x = 0 \\ p\dfrac{1}{\left[1 + \tanh\left(\dfrac{\pi\mu}{2\sqrt{3}\,\sigma}\right)\right]} \operatorname{sech}^2\left(\dfrac{\pi(x-\mu)}{2\sqrt{3}\,\sigma}\right) & , x > 0 \\ 0 & , \text{otherwise,} \end{cases} \tag{13}$$

where:

μ = mean of all non-zero demand observations,

σ = standard deviation of all non-zero demand observations.

After integrating (13), the complementary cumulative distribution function may be written as:

$$H(x) = p\left\{1 - \frac{1}{\left[1 + \tanh\left(\frac{\pi\mu}{2\sqrt{3}\sigma}\right)\right]}\right\}\left[\tanh\left(\frac{\pi\mu}{2\sqrt{3}\sigma}\right) + \tanh\left(\frac{\pi(x-\mu)}{2\sqrt{3}\sigma}\right)\right], \quad (14)$$

for $x \geq 0$

The inverse cumulative distribution is:

$$x = \begin{cases} 0 & , H(x) \geq p \\ \mu + \frac{\sqrt{3}\sigma}{\pi}\ln\left\{\frac{2p}{H(x)\left[1 + \tanh\left(\frac{\pi\mu}{2\sqrt{3}\sigma}\right)\right]} - 1\right\} & , H(x) < p \end{cases} \quad (15)$$

The parameters are estimated from the sample data in the usual manner:

$\hat{p}$ = number of non-zero demand observations in sample divided by sample size,

$\hat{\mu} = \bar{x}$, where $\bar{x}$ is the sample mean of all non-zero demand observations,

$\hat{\sigma} = s$, where s is the sample standard deviation of all non-zero demand observations.

IV. EVALUATION PROCEDURE

A. THE DATA

In order to test the validity of the proposed models, samples of actual demand were obtained from the Operations Analysis Department at FMSO. The data, accumulated from the demand history files of the Aviation Supply Office, was originally used as input data for the 5A (Aviation Afloat and Ashore Allowance Analyzer) [Ref. 7]. The data consists of 1587 consumable 1R cog items and 2892 non-program-related[9] repairable 2R cog items. The information for each item is contained in one master record and several subrecords. The master record contains identifying information on each item such as the national stock number, replacement price, etc. Each subrecord contains up to forty-six demand records, each including the demand quantity and the day of the demand. A complete record layout is contained in Appendix E. To ease processing, the Julian Dates of the original demands were replaced by FMSO with a sequential date ranging from 1 to 1500 representing approximately four years of available history.

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[9]See Appendix A for definition of "program related" items.

For this application, additional screening was conducted to group the demands into thirty-day buckets, thus providing a demand time series of forty-eight observations. Further editing was required prior to analysis to remove the negative demands noted for many of the items. It was assumed that the negative observations were the result of cancellations of previous demands which were never filled. Therefore, whenever a negative demand was encountered, the demand series was searched backwards for the first positive demand equal to or larger than the negative demand. If such a demand was found, the positive demand quantity was reduced by the absolute value of the negative quantity and the negative quantity was set equal to zero. If no offsetting positive demand was found, the negative demand quantity was still set equal to zero. The resulting edited demand time series was used as the base for all further analysis. Two additional demand series were created from the edited series:

1. A demand series of only the non-zero demands,
2. A demand series of the log of the non-zero demands.

The sample mean and sample standard deviation for each of the three demand series were computed[10] and the group, representing one line item, was accepted for futher processing if:

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[10]Stationarity was assumed for the demand time series under study. If a trend were in fact present, the resampling technique effectively would eliminate it by shuffling the observed demands into random order.

1. There were at least two non-zero demands, and
2. The standard deviation of all three series was non-zero.

Once accepted, the sample mean and/or sample standard deviation of the appropriate series was used to estimate the value of x, such that for a given probability, demand will not exceed that value of x. The values of x were computed for each model tested at several probabilities from the inverse cumulative probability functions. Table 2 displays the sample mean and/or sample standard deviation required for each model investigated.

B. THE RESAMPLING PROCEDURE

The resampling procedure used was chosen over the more traditional methods discussed in Chapter II because it provides a method of comparing a theoretical distribution to the sample data at specified probabilities and is applicable in cases such as this where there is a paucity of data available for analysis. This procedure is similar to the "bootstrap" procedure used by Efron [Ref. 8]. The idea behind this procedure is to randomly sample with replacement the series of available data to create additional pseudo-samples that possess the same statistical properties as the original sample. The desired statistical property can then be estimated from

Table 2: Required Data for Inverse Computations

	MODEL	MEAN	STANDARD DEVIATION
1.	Exponential	A	N/R
2.	Normal	A	A
3.	Poisson	A	N/R
4.	Negative Binomial	A	A
5.	Lognormal	C	C
6.	Logistic	A	A
7.	La Place	A	A
8.	Bernoulli-exponential	B	N/R
9.	Bernoulli-lognormal	C	C
10.	Bernoulli-logistic	B	B

Where:

A is from edited demand series
B is from non-zero demand series
C is from log of non-zero demand series
N/R indicates parameter Not Required for this model

each pseudo sample. With a sufficient number of repetitions, the distribution of the estimated property is known to be normal from the central limit theorem with mean equal to the population mean and standard deviation equal to the population standard deviation divided by the number of repetitions.

Thus by the use of the resampling procedure, it is possible to study theoretical distributions at points in their

right hand tail area, which is the region of most interest in computing safety levels in inventory models. The resampling procedure, as applied in this study, is a relatively straightforward application of the following technique:

1. The edited series for each sample of n demand observations is treated as the population from which the random samples are drawn.
2. A random sample of size n is drawn from this population, creating a pseudo sample. The pseudo sample is not a permutation of the original population since the sample values are selected with replacement from the original population. This is easily accomplished in a computer by generating n uniform random integers in the range from 1 to n, and using these numbers as subscripts to select the sample from the original population.
3. The pseudo sample percentile is computed for each repetition from (16) and is compared to the theoretical percentile.

$$\hat{\rho} = (\text{number of demand observations} \leq x)\ /\ n \qquad (16)$$

The various values of x are computed from the inverse cumulative probability functions for each model evaluated.

Under the null hypothesis that the sample demands come from a particular probability distribution, the expected value of $\hat{\rho}$ should equal ρ, the theoretical percentile:

$$E[\hat{\rho}] = \rho \text{ under } H_0 \tag{17}$$

and the distribution of ρ should be Normal$(\rho, \sigma^2/n)$. The percentile ρ is estimated as above and σ^2 is estimated from standard binomial results as:

$$Var[\hat{\rho}] = \hat{\rho}(1 - \hat{\rho}) / n. \tag{18}$$

C. MEASURES OF EFFECTIVENESS

When the above precedure is executed at several theoretical percentiles, the fit of the sample data may be compared at several locations in the theoretical distribution and the overall fit evaluated. Several common measures of effectiveness are available to provide a criteria on which to judge the success or failure of a model. Some of those available are:

1. Maximum absolute deviation,
2. Mean squared error,
3. Algebraic sum of errors.

Method three was eliminated since errors may offset one another resulting in a seemingly good fit, but in reality a

very poor fit. Method one is potentially a good way to evaluate error, but is generally not as sensitive as method two. Therefore, the mean squared error of the pseudo sample percentile estimates was chosen as the measure of effectiveness of choice and was accumulated as:

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (\hat{\rho}_i - \rho_i)^2. \tag{19}$$

D. COMPUTER VALIDATION

The computer programs for this analysis were written in FORTRAN IV for execution on the IBM 3033 attached processor (System 370) computer installed at the Naval Postgraduate School. The programs are batch oriented since the volume of data available prevented storage directly on the user's private disk space. The data tapes received from FMSO were loaded onto the system mass storage device and when required, all or portions of the data were transferred to the Virtual Machine (VM) facility for processing. Two programs were written, the first to copy the data to make it accessible to the VM system and the second to conduct the actual analysis. A complete listing of the FORTRAN source code is available in Appendix F. Each program or subprogram contains a documentation block at the top that provides a complete description of

the program's purpose, a definition of variables used and a list of user-written subroutines and functions required.

The random number generator used is a part of the LLRANDOM II series [Ref. 9] developed at the Naval Postgraduate School. The programs and subprograms were thoroughly tested both independently and as a unit using known data. The results were verified by hand held calculator and in all cases agreed with the program output.

V. RESULTS

The results of the distributional analysis conducted on the ten distribution models listed in Table 2 are tabulated in Appendix B for 1R consumable items. Appendix B is divided into the three demand classes listed in Table 1, and then each class is divided by distribution and percentile within each distribution. The results for the 2R repairable items are presented in a similar format in Appendix C.

The consistency of the resampling method with varying data was evaluated by testing independent sections of the consumable and repairable data sets and comparing the results. In both cases, independent subsamples were created by sampling every third item but changing the initial item sampled from the first to the third item. The analysis, not presented here, showed that the resampling method produced consistent results over all of the subsamples tested. The total mean square error[11] of the distributions in each subsample varied by no more than 20% from the total mean square error values obtained from the entire sample population as listed in Appendix B and Appendix C. The relative standing

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[11]Total mean square error is the sum of the mean square errors computed at the 75th, 80th, 85th, 90th and 95th percentiles.

of each distribution was not altered in any of the subsamples.

The effect of the number of pseudo sample repetitions was also studied. The results proved similar while varying the number of repetitions from twenty to fifty. The standard deviation of the percentile estimates decreased as the number of repetitions increased. The total mean square error for each trial varied randomly but remained within 10% of the overall values for each distribution with more repetitions producing consistently smaller variations. For the data analysis production runs, forty was chosen as the number of resampling repetitions as a trade off between computer run time and minimizing variance.

A. LOW-DEMAND ITEMS

A summary of the total mean squared error figures for the models providing the best fit or the smallest total mean squared error is provided in Table 3 for the low-demand items. The best models for the low demand items all had very small total mean squared errors. This can be attributed to the very low probability of a demand ever being greater than one and to the integer sampling plan for accumulating the Bernoulli trials. For the integer results of the Poisson and negative binomial distributions, the estimated percentile

TABLE 3: LOW-DEMAND ITEMS,

Total Mean Squared Error (10^{-4})

	Poisson	Negative binomial	Compound
Consumable 1R	5.64	7.86	10.77
Repairable 2R	5.86	7.57	12.91

will closely match the theoretical percentile unless the number of Bernoulli successes is less than the expected number of successes for a given theoretical percentile. This produces a conservative result that will tend to overstate the quantity of stock required by one unit to provide a stated level of protection, but in the case of the small quantities involved, is not an undesirable property. The compound distributions, as a group, gave the same total mean squared error for much the same reasons. The probability of a non-zero demand is very small for the low-demand items, generally less than 0.05, thus these models tend to predict a quantity of zero for all percentiles. The resampling procedure and the sampling plan again provide a conservative result.

B. MEDIUM-DEMAND ITEMS

A summary of the total mean squared error figures for the models providing the best fit or the smallest total mean

squared error is provided in Table 4 for the medium-demand items. Here again the predicted demand for the various percentiles is small. The resampling technique still gives the

TABLE 4: MEDIUM-DEMAND ITEMS,

Total Mean Squared Error (10^{-4})

	Negative binomial	Poisson	Bernoulli-lognormal
Consumable 1R	28.55	48.28	80.60
Repairable 2R	29.14	44.84	112.44

edge to the integer distributions. The compound distributions have begun to show some differences among themselves, but as a group are clearly better than any of the continuous simple distributions. The magnitude of the total mean squared error is significantly larger than for the low-demand class primarily due to the larger allowable total demand range defining the medium-demand class.

C. HIGH-DEMAND ITEMS

The summary total mean squared error data for the high-demand items are listed in Table 5. With the unlimited range of mean demands in this class, the higher total mean squared errors are expected. The compound models have come into

TABLE 5: HIGH-DEMAND ITEMS,

Total Mean Squared Error (10^{-4})

	Bernoulli-lognormal	Bernoulli-exponential	Exponential
Consumable 1R	168.25	188.90	298.21
Repairable 2R	183.77	221.80	289.01

their own in this class with the Bernoulli-lognormal yielding a total mean squared error less than one half that of the normal distribution. The second best distribution, the Bernoulli-exponential, gives a total mean squared error only slightly higher than the Bernoulli-lognormal.

VI. CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY

This distributional analysis, conducted using a completely different experimental procedure than prior analyses, provides satisfyingly similar results for the low and medium demand categories in showing that the Poisson distribution gives the best fit for the low-demand category of items and that the negative binomial distribution gives the best fit for the medium-demand category of items. The class of compound distributions, as a whole, gave good fits in the low- and medium-demand categories, though the total mean squared error was two to four times as large as that of the Poisson or negative binomial distribution.

The analysis indicated that there are several distributions which provide better results than the normal distribution for high-demand items. The Bernoulli-lognormal distribution consistently provided total mean squared errors approximately one half that of the normal distribution, indicating a superior fit in the right-hand tail area of the distribution. Other distributions giving good fits were the Bernoulli-exponential and the standard exponential, both yielding total mean squared errors less than that of the normal distribution.

B. RECOMMENDATIONS

The results of this study indicate the Navy should continue to use the Poisson and negative binomial distributions as the models for the low- and medium-demand classes respectively. Specifically, the Aviation Supply Office should re-evaluate its position on the use of the normal distribution for all demand classes and revert to the Poisson and negative binomial distributions for low- and medium-demand classes as before. The normal distribution tends to inflate the stock required for low- and medium-demand items for a specified level of protection resulting in increased safety levels and excessive dollar investment.

The Navy should consider replacing the normal distribution model used for high-demand items with the compound Bernoulli-lognormal distribution. If the Bernoulli-lognormal distribution is considered computationally too difficult for a large inventory system, the Bernoulli-exponential could be used in its place with little loss of effectiveness.

C. ADDITIONAL RESEARCH

Follow on research in the following areas may improve and expand upon the results presented above:

1. Optimize the divisions of demand categories (i.e. low, medium, high) with the possible use of the probability

of a non-zero demand as an element in the determination,

2. Implement the Bernoulli-lognormal model in FMSO's 5A simulator to establish the effect of the change on the entire supply system,
3. Investigate the implications of the assumption that the demand process is stationary,
4. Evaluate other compound distributions, such as the Bernoulli-log-logistic, which is particularly appealing analytically.

APPENDIX A

GLOSSARY OF TERMS

Program Related Item: An item of stock whose demand can be predicted from the value of a specific Navy program, i.e., flying hours, steaming hours, etc.

Reorder Point: The on hand stock quantity that when reached, triggers an order for replenishment of stock material. The reorder Point is the expected demand during procurement lead-time plus the safety level.

Risk: Probability of a stockout during leadtime.

Safety Level: The quantity of material which is required to be on hand to permit continued operation in the event of minor interuptions on normal replenishment or unpredictable fluctuations in demand. The safety level determined is structured so as to minimize time-weighted, essentiality-weighted requisitions short.

Stockout: A condition that exists when the on hand inventory is insufficient to fill the current demand requirements.

System Performance: A subjective measure maximized when the time-weighted, essentiality-weighted requisitions short is minimized. Time weighting is the consideration of the average number of days delay in the availability of material, essentiality-weighting is the consideration of the relative essentiality of each item. Requisitions short are requisitions for which material is not available.

APPENDIX B

CONSUMABLE ITEM TABULATED RESULTS

40 REPETITIONS FOR EACH OF 61 PSEUDO SAMPLES

LOW DEMAND RANGE: 0.0 < D < 1.0 PER YEAR

DISTRIBUTION: POISSON

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.750000	0.812500	0.854006	0.913924	0.956069
STD DEV	0.008766	0.007902	0.007148	0.005678	0.004149
MSE	0.0	0.000156	0.000018	0.000290	0.000100

TOTAL MSE: 5.64E-04

DISTRIBUTION: NEGATIVE BINOMIAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.750000	0.812500	0.854006	0.913924	0.948994
STD DEV	0.008766	0.007902	0.007148	0.005678	0.004454
MSE	0.0	0.000156	0.000018	0.000290	0.000322

TOTAL MSE: 7.86E-04

40 REPETITIONS FOR EACH OF 61 PSEUDO SAMPLES

LOW DEMAND RANGE: 0.0 < D < 1.0 PER YEAR

DISTRIBUTION: NORMAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.953221	0.953221	0.953221	0.953221	0.953221
STD DEV	0.004275	0.004275	0.004275	0.004275	0.004275
MSE	0.042319	0.024488	0.011662	0.003835	0.001006

TOTAL MSE: 833.09E-04

DISTRIBUTION: EXPONENTIAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.953221	0.953221	0.953221	0.953221	0.953221
STD DEV	0.004275	0.004275	0.004275	0.004275	0.004275
MSE	0.042319	0.024488	0.011662	0.003835	0.001006

TOTAL MSE: 833.09E-04

DISTRIBUTION: LOGNORMAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.978404	0.995246	0.995246	1.000000	1.000000
STD DEV	0.002943	0.001392	0.001392	0.0	0.0
MSE	0.052629	0.038292	0.021267	0.009998	0.002500

TOTAL MSE: 1246.85E-04

40 REPETITIONS FOR EACH OF 61 PSEUDO SAMPLES

LOW DEMAND RANGE: 0.0 < D < 1.0 PER YEAR

DISTRIBUTION: LAPLACE

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.953221	0.953221	0.953221	0.953221	0.953221
STD DEV	0.004275	0.004275	0.004275	0.004275	0.004275
MSE	0.042319	0.024488	0.011662	0.003835	0.001006

TOTAL MSE: 833.09E-04

DISTRIBUTION: LOGISTIC

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.953221	0.953221	0.953221	0.953221	0.953221
STD DEV	0.004275	0.004275	0.004275	0.004275	0.004275
MSE	0.042319	0.024488	0.011662	0.003835	0.001006

TOTAL MSE: 833.09E-04

DISTRIBUTION: BERNOULLI-EXPONENTIAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.750000	0.812500	0.854006	0.913924	0.944939
STD DEV	0.008766	0.007902	0.007148	0.005678	0.004618
MSE	0.0	0.000156	0.000018	0.000290	0.000613

TOTAL MSE: 10.77E-04

40 REPETITIONS FOR EACH OF 61 PSEUDO SAMPLES

LOW DEMAND RANGE: 0.0 < D < 1.0 PER YEAR

DISTRIBUTION: BERNOULLI-LOGNORMAL

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
	THEORETICAL PERCENTILES (P)				
MEAN	0.750000	0.812500	0.854006	0.913924	0.944939
STD DEV	0.008766	0.007902	0.007148	0.005678	0.004618
MSE	0.0	0.000156	0.000018	0.000290	0.000613

TOTAL MSE: 10.77E-04

DISTRIBUTION: BERNOULLI-LOGISTIC

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
	THEORETICAL PERCENTILES (P)				
MEAN	0.750000	0.812500	0.854006	0.913924	0.944939
STD DEV	0.008766	0.007902	0.007148	0.005678	0.004618
MSE	0.0	0.000156	0.000018	0.000290	0.000613

TOTAL MSE: 10.77E-04

40 REPETITIONS FOR EACH OF 753 PSEUDO SAMPLES

MEDIUM DEMAND RANGE: 1.0 < D < 20.0 PER YEAR

DISTRIBUTION: POISSON

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
	THEORETICAL PERCENTILES (P)				
MEAN	0.748131	0.807719	0.843932	0.893923	0.925303
STD DEV	0.002501	0.002271	0.002091	0.001774	0.001515
MSE	0.000138	0.000398	0.000603	0.001423	0.002266

TOTAL MSE: 48.28E-04

DISTRIBUTION: NEGATIVE BINOMIAL

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
	THEORETICAL PERCENTILES (P)				
MEAN	0.746099	0.805430	0.843400	0.900210	0.943628
STD DEV	0.002508	0.002281	0.002094	0.001727	0.001329
MSE	0.000332	0.000522	0.000589	0.000807	0.000606

TOTAL MSE: 28.55E-04

DISTRIBUTION: NORMAL

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
	THEORETICAL PERCENTILES (P)				
MEAN	0.888030	0.896699	0.907882	0.920152	0.935573
STD DEV	0.001817	0.001754	0.001666	0.001562	0.001415
MSE	0.024771	0.014168	0.007327	0.003509	0.002400

TOTAL MSE: 521.74E-04

40 REPETITIONS FOR EACH OF 753 PSEUDO SAMPLES

MEDIUM DEMAND RANGE: 1.0 < D < 20.0 PER YEAR

DISTRIBUTION: <u>EXPONENTIAL</u>

	THEORETICAL PERCENTILES (P)				
<u>ESTIMATES</u>:	<u>0.75</u>	<u>0.80</u>	<u>0.85</u>	<u>0.90</u>	<u>0.95</u>
MEAN	0.852981	0.861003	0.871591	0.884311	0.898352
STD DEV	0.002040	0.001993	0.001928	0.001843	0.001741
MSE	0.020357	0.011779	0.006819	0.004953	0.006210
TOTAL MSE:	501.19E-04				

DISTRIBUTION: <u>LOGNORMAL</u>

	THEORETICAL PERCENTILES (P)				
<u>ESTIMATES</u>:	<u>0.75</u>	<u>0.80</u>	<u>0.85</u>	<u>0.90</u>	<u>0.95</u>
MEAN	0.953018	0.962552	0.970638	0.980623	0.990731
STD DEV	0.001219	0.001094	0.000973	0.000794	0.000552
MSE	0.043846	0.028367	0.015977	0.007411	0.002040
TOTAL MSE:	976.41E-04				

DISTRIBUTION: <u>LAPLACE</u>

	THEORETICAL PERCENTILES (P)				
<u>ESTIMATES</u>:	<u>0.75</u>	<u>0.80</u>	<u>0.85</u>	<u>0.90</u>	<u>0.95</u>
MEAN	0.884254	0.893767	0.904733	0.918110	0.936078
STD DEV	0.001843	0.001775	0.001692	0.001580	0.001409
MSE	0.024079	0.013865	0.007191	0.003573	0.002363
TOTAL MSE:	510.72E-04				

40 REPETITIONS FOR EACH OF 753 PSEUDO SAMPLES

MEDIUM DEMAND RANGE: 1.0 < D < 20.0 PER YEAR

DISTRIBUTION: LOGISTIC

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.875036	0.886687	0.896908	0.913037	0.934834
STD DEV	0.001905	0.001826	0.001752	0.001624	0.001422
MSE	0.022871	0.013193	0.006955	0.003690	0.002447
TOTAL MSE:	491.56E-04				

DISTRIBUTION: BERNOULLI-EXPONENTIAL

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.738072	0.795951	0.840430	0.896563	0.950685
STD DEV	0.002533	0.002322	0.002110	0.001755	0.001248
MSE	0.002132	0.002338	0.002267	0.002224	0.001327
TOTAL MSE:	102.88E-04				

DISTRIBUTION: BERNOULLI-LOGNORMAL

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.749344	0.809159	0.849391	0.903886	0.948263
STD DEV	0.002497	0.002264	0.002061	0.001698	0.001276
MSE	0.001456	0.001751	0.001730	0.001866	0.001258
TOTAL MSE:	80.60E-04				

40 REPETITIONS FOR EACH OF 753 PSEUDO SAMPLES

MEDIUM DEMAND RANGE: 1.0 < D < 20.0 PER YEAR

DISTRIBUTION: BERNOULLI-LOGISTIC

	THEORETICAL PERCENTILES (P)				
ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.760006	0.821542	0.864176	0.916219	0.955416
STD DEV	0.002461	0.002206	0.001974	0.001596	0.001189
MSE	0.002256	0.002499	0.002304	0.002030	0.001241

TOTAL MSE: 103.30E-04

40 REPETITIONS FOR EACH OF 429 PSEUDO SAMPLES

HIGH DEMAND RANGE: 20.0 < D PER YEAR

DISTRIBUTION: POISSON

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.689018	0.730300	0.753320	0.780038	0.800697
STD DEV	0.003534	0.003388	0.003291	0.003162	0.003050
MSE	0.024843	0.027146	0.033935	0.042154	0.051388

TOTAL MSE: 1794.66E-04

DISTRIBUTION: NEGATIVE BINOMIAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.724173	0.747130	0.791313	0.848041	0.896526
STD DEV	0.003412	0.003318	0.003102	0.002740	0.002325
MSE	0.003047	0.024647	0.026816	0.029773	0.031637

TOTAL MSE: 1159.20E-04

DISTRIBUTION: NORMAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.850264	0.871100	0.890154	0.910670	0.934248
STD DEV	0.002724	0.002558	0.002387	0.002177	0.001892
MSE	0.016622	0.010312	0.005815	0.003155	0.002227

TOTAL MSE: 381.32E-04

40 REPETITIONS FOR EACH OF 429 PSEUDO SAMPLES

HIGH DEMAND RANGE: 20.0 < D PER YEAR

DISTRIBUTION: EXPONENTIAL

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.784610	0.814275	0.846685	0.881207	0.918702
STD DEV	0.003138	0.002969	0.002750	0.002470	0.002086
MSE	0.009534	0.006885	0.005219	0.004384	0.003800

TOTAL MSE: 298.21E-04

DISTRIBUTION: LOGNORMAL

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.855867	0.886448	0.918614	0.949144	0.978288
STD DEV	0.002681	0.002422	0.002087	0.001677	0.001113
MSE	0.019028	0.013031	0.008113	0.004389	0.001639

TOTAL MSE: 462.00E-04

DISTRIBUTION: LAPLACE

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.843416	0.863600	0.885612	0.907473	0.934956
STD DEV	0.002774	0.002620	0.002430	0.002212	0.001883
MSE	0.015775	0.009709	0.005680	0.003248	0.002167

TOTAL MSE: 365.79E-04

40 REPETITIONS FOR EACH OF 429 PSEUDO SAMPLES

HIGH DEMAND RANGE: 20.0 < D PER YEAR

DISTRIBUTION: LOGISTIC

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.822132	0.846791	0.871709	0.900110	0.933755
STD DEV	0.002919	0.002750	0.002553	0.002289	0.001899
MSE	0.013721	0.008841	0.005633	0.003523	0.002259

TOTAL MSE: 339.76E-04

DISTRIBUTION: BERNOULLI-EXPONENTIAL

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.759754	0.816600	0.867492	0.914225	0.955101
STD DEV	0.003261	0.002954	0.002588	0.002138	0.001581
MSE	0.005828	0.005034	0.003977	0.002618	0.001432

TOTAL MSE: 188.90E-04

DISTRIBUTION: BERNOULLI-LOGNORMAL

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.745078	0.797614	0.847684	0.901963	0.953670
STD DEV	0.003327	0.003067	0.002743	0.002270	0.001605
MSE	0.004768	0.004403	0.003632	0.002613	0.001409

TOTAL MSE: 168.25E-04

40 REPETITIONS FOR EACH OF 429 PSEUDO SAMPLES

HIGH DEMAND RANGE: 20.0 < D PER YEAR

DISTRIBUTION: BERNOULLI-LOGISTIC

	THEORETICAL PERCENTILES (P)				
ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.828539	0.864722	0.892571	0.921326	0.947036
STD DEV	0.002877	0.002611	0.002364	0.002055	0.001710
MSE	0.012517	0.008862	0.005485	0.003020	0.001538

TOTAL MSE: 314.23E-04

APPENDIX C

REPAIRABLE ITEM TABULATED RESULTS

40 REPETITIONS FOR EACH OF 137 PSEUDO SAMPLES

LOW DEMAND RANGE: 0.0 < D < 1.0 PER YEAR

DISTRIBUTION: POISSON

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.750000	0.812496	0.853613	0.912594	0.955936
STD DEV	0.005849	0.005273	0.004775	0.003815	0.002772
MSE	0.0	0.000156	0.000019	0.000311	0.000099

TOTAL MSE: 5.86E-04

DISTRIBUTION: NEGATIVE BINOMIAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.750000	0.812496	0.853613	0.912594	0.950753
STD DEV	0.005849	0.005273	0.004775	0.003815	0.002923
MSE	0.0	0.000156	0.000019	0.000311	0.000270

TOTAL MSE: 7.57E-04

40 REPETITIONS FOR EACH OF 137 PSEUDO SAMPLES

LOW DEMAND RANGE: 0.0 < D < 1.0 PER YEAR

DISTRIBUTION: NORMAL

	THEORETICAL PERCENTILES (P)				
ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.949164	0.949164	0.949164	0.949164	0.949164
STD DEV	0.002967	0.002967	0.002967	0.002967	0.002967
MSE	0.040918	0.023465	0.011016	0.003568	0.001118

TOTAL MSE: 800.84E-04

DISTRIBUTION: EXPONENTIAL

	THEORETICAL PERCENTILES (P)				
ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.949164	0.949164	0.949164	0.949164	0.949164
STD DEV	0.002967	0.002967	0.002967	0.002967	0.002967
MSE	0.040918	0.023465	0.011016	0.003568	0.001118

TOTAL MSE: 800.84E-04

DISTRIBUTION: LOGNORMAL

	THEORETICAL PERCENTILES (P)				
ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.978874	0.991028	0.991028	1.000000	1.000000
STD DEV	0.001943	0.001274	0.001274	0.0	0.0
MSE	0.052955	0.036815	0.020201	0.009996	0.002500

TOTAL MSE: 1224.66E-04

40 REPETITIONS FOR EACH OF 137 PSEUDO SAMPLES

LOW DEMAND RANGE: 0.0 < D < 1.0 PER YEAR

DISTRIBUTION: LAPLACE

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.949164	0.949164	0.949164	0.949164	0.949164
STD DEV	0.002967	0.002967	0.002967	0.002967	0.002967
MSE	0.040918	0.023465	0.011016	0.003568	0.001118

TOTAL MSE: 800.84E-04

DISTRIBUTION: LOGISTIC

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.949164	0.949164	0.949164	0.949164	0.949164
STD DEV	0.002967	0.002967	0.002967	0.002967	0.002967
MSE	0.040918	0.023465	0.011016	0.003568	0.001118

TOTAL MSE: 800.84E-04

DISTRIBUTION: BERNOULLI-EXPONENTIAL

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.750000	0.812496	0.853613	0.912594	0.942635
STD DEV	0.005849	0.005273	0.004775	0.003815	0.003141
MSE	0.0	0.000156	0.000019	0.000311	0.000804

TOTAL MSE: 12.91E-04

40 REPETITIONS FOR EACH OF 137 PSEUDO SAMPLES

LOW DEMAND RANGE: 0.0 < D < 1.0 PER YEAR

DISTRIBUTION: BERNOULLI-LOGNORMAL

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.750000	0.812496	0.853613	0.912594	0.942635
STD DEV	0.005849	0.005273	0.004775	0.003815	0.003141
MSE	0.0	0.000156	0.000019	0.000311	0.000804

TOTAL MSE: 12.91E-04

DISTRIBUTION: BERNOULLI-LOGISTIC

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.750000	0.812496	0.853613	0.912594	0.942635
STD DEV	0.005849	0.005273	0.004775	0.003815	0.003141
MSE	0.0	0.000156	0.000019	0.000311	0.000804

TOTAL MSE: 12.91E-04

40 REPETITIONS FOR EACH OF 1487 PSEUDO SAMPLES

MEDIUM DEMAND RANGE: 1.0 < D < 20.0 PER YEAR

DISTRIBUTION: POISSON

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.747034	0.806323	0.843928	0.894732	0.929457
STD DEV	0.001782	0.001620	0.001488	0.001258	0.001050
MSE	0.000238	0.000507	0.000582	0.001321	0.001836

TOTAL MSE: 44.84E-04

DISTRIBUTION: NEGATIVE BINOMIAL

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.744688	0.804450	0.843869	0.900645	0.944953
STD DEV	0.001788	0.001626	0.001488	0.001227	0.000935
MSE	0.000444	0.000598	0.000533	0.000788	0.000551

TOTAL MSE: 29.14E-04

DISTRIBUTION: NORMAL

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.861474	0.875334	0.890316	0.905783	0.927155
STD DEV	0.001416	0.001354	0.001281	0.001198	0.001066
MSE	0.019444	0.011363	0.006084	0.003600	0.002944

TOTAL MSE: 434.35E-04

40 REPETITIONS FOR EACH OF 1487 PSEUDO SAMPLES

MEDIUM DEMAND RANGE: 1.0 < D < 20.0 PER YEAR

DISTRIBUTION: EXPONENTIAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.823217	0.834022	0.845365	0.864271	0.886389
STD DEV	0.001564	0.001526	0.001482	0.001404	0.001301
MSE	0.016435	0.010140	0.007542	0.006691	0.007993

TOTAL MSE: 488.02E-04

DISTRIBUTION: LOGNORMAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.946618	0.955351	0.964288	0.975138	0.987135
STD DEV	0.000922	0.000847	0.000761	0.000638	0.000462
MSE	0.041761	0.026454	0.014803	0.006692	0.001890

TOTAL MSE: 916.00E-04

DISTRIBUTION: LAPLACE

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.856232	0.870161	0.885855	0.902481	0.928262
STD DEV	0.001439	0.001378	0.001304	0.001216	0.001058
MSE	0.018823	0.011026	0.006113	0.003736	0.002851

TOTAL MSE: 425.48E-04

40 REPETITIONS FOR EACH OF 1487 PSEUDO SAMPLES

MEDIUM DEMAND RANGE: 1.0 < D < 20.0 PER YEAR

DISTRIBUTION: LOGISTIC

	THEORETICAL PERCENTILES (P)				
ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.845409	0.858922	0.875792	0.896303	0.925823
STD DEV	0.001482	0.001427	0.001352	0.001250	0.001075
MSE	0.017690	0.010603	0.006269	0.004086	0.003093
TOTAL MSE:	417.42E-04				

DISTRIBUTION: BERNOULLI-EXPONENTIAL

	THEORETICAL PERCENTILES (P)				
ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.730065	0.787917	0.834508	0.893026	0.954267
STD DEV	0.001820	0.001676	0.001524	0.001267	0.000857
MSE	0.003437	0.003608	0.003449	0.003021	0.001465
TOTAL MSE:	149.79E-04				

DISTRIBUTION: BERNOULLI-LOGNORMAL

	THEORETICAL PERCENTILES (P)				
ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.750179	0.808666	0.852134	0.904194	0.949477
STD DEV	0.001775	0.001613	0.001455	0.001207	0.000898
MSE	0.002226	0.002681	0.002600	0.002327	0.001410
TOTAL MSE:	112.44E-04				

40 REPETITIONS FOR EACH OF 1487 PSEUDO SAMPLES

MEDIUM DEMAND RANGE: 1.0 < D < 20.0 PER YEAR

DISTRIBUTION: BERNOULLI-LOGISTIC

	THEORETICAL PERCENTILES (P)				
ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.762406	0.822694	0.866844	0.915851	0.953988
STD DEV	0.001745	0.001566	0.001393	0.001138	0.000859
MSE	0.003183	0.003523	0.003107	0.002468	0.001379

TOTAL MSE: 136.61E-04

40 REPETITIONS FOR EACH OF 356 PSEUDO SAMPLES

HIGH DEMAND RANGE: 20.0 < D PER YEAR

DISTRIBUTION: POISSON

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.695829	0.735554	0.762330	0.791447	0.821025
STD DEV	0.003855	0.003696	0.003567	0.003405	0.003212
MSE	0.022812	0.026569	0.032141	0.038638	0.044655

TOTAL MSE: 1648.16E-04

DISTRIBUTION: NEGATIVE BINOMIAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.722861	0.755975	0.799852	0.857560	0.904930
STD DEV	0.003751	0.003599	0.003353	0.002929	0.002458
MSE	0.005285	0.024724	0.026795	0.028699	0.030113

TOTAL MSE: 1156.16E-04

DISTRIBUTION: NORMAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.836334	0.859810	0.883941	0.905503	0.932035
STD DEV	0.003100	0.002909	0.002684	0.002451	0.002109
MSE	0.014669	0.009278	0.005429	0.003190	0.002319

TOTAL MSE: 348.85E-04

40 REPETITIONS FOR EACH OF 356 PSEUDO SAMPLES

HIGH DEMAND RANGE: 20.0 < D PER YEAR

DISTRIBUTION: EXPONENTIAL

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.780940	0.818187	0.856974	0.898153	0.935255
STD DEV	0.003466	0.003232	0.002934	0.002535	0.002062
MSE	0.009578	0.007296	0.005229	0.003830	0.002968

TOTAL MSE: 289.01E-04

DISTRIBUTION: LOGNORMAL

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.819119	0.855337	0.895522	0.933281	0.971445
STD DEV	0.003226	0.002948	0.002563	0.002091	0.001396
MSE	0.013003	0.008704	0.005985	0.003420	0.001454

TOTAL MSE: 325.65E-04

DISTRIBUTION: LAPLACE

ESTIMATES:	THEORETICAL PERCENTILES (P) 0.75	0.80	0.85	0.90	0.95
MEAN	0.827652	0.852088	0.877334	0.902360	0.932870
STD DEV	0.003165	0.002975	0.002749	0.002487	0.002097
MSE	0.013659	0.008798	0.005346	0.003302	0.002267

TOTAL MSE: 333.71E-04

40 REPETITIONS FOR EACH OF 356 PSEUDO SAMPLES

HIGH DEMAND RANGE: 20.0 < D PER YEAR

DISTRIBUTION: LOGISTIC

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.805812	0.833134	0.860882	0.892308	0.930748
STD DEV	0.003315	0.003125	0.002900	0.002598	0.002128
MSE	0.012232	0.008347	0.005666	0.003803	0.002397

THEORETICAL PERCENTILES (P)

TOTAL MSE: 324.44E-04

DISTRIBUTION: BERNOULLI-EXPONENTIAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.770942	0.829521	0.877192	0.921468	0.960362
STD DEV	0.003522	0.003151	0.002750	0.002254	0.001635
MSE	0.007108	0.005916	0.004548	0.003092	0.001516

TOTAL MSE: 221.80E-04

DISTRIBUTION: BERNOULLI-LOGNORMAL

THEORETICAL PERCENTILES (P)

ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.746108	0.797150	0.847197	0.901039	0.953617
STD DEV	0.003647	0.003370	0.003015	0.002502	0.001762
MSE	0.005655	0.004764	0.003680	0.002809	0.001469

TOTAL MSE: 183.77E-04

40 REPETITIONS FOR EACH OF 356 PSEUDO SAMPLES

HIGH DEMAND RANGE: 20.0 < D PER YEAR

DISTRIBUTION: BERNOULLI-LOGISTIC

	THEORETICAL PERCENTILES (P)				
ESTIMATES:	0.75	0.80	0.85	0.90	0.95
MEAN	0.829834	0.861163	0.886755	0.914062	0.941856
STD DEV	0.003149	0.002898	0.002656	0.002349	0.001961
MSE	0.013074	0.008963	0.005495	0.003143	0.001722

TOTAL MSE: 323.98E-04

APPENDIX D

ESTIMATION OF LOGNORMAL MEAN AND VARIANCE FROM UNTRANSFORMED DATA

The usual method of estimating the sample mean and variance of a random sample assumed to come from a lognormal distribution is to take the log transform of the data and then compute the standard method of moments estimates. This is time consuming on a computer as the logarithm must be taken for each observation in the sample. Formulae to compute the mean and variance of the untransformed data from the mean and variance of the transformed data can be derived for the lognormal distribution.

If Y is distributed normal(μ_Y, σ_Y^2), then X is distributed lognormal if Y = ln(X). The mean of X [Ref. 10] is expressed as:

$$\mu_X = e^{\mu_Y + \frac{1}{2}\sigma_Y^2} \tag{20}$$

and the variance as:

$$\sigma_X^2 = e^{2(\mu_Y + \frac{1}{2}\sigma_Y^2)}\left(e^{\sigma_Y^2} - 1\right). \tag{21}$$

Equations (20) and (21) are the reverse of that required in this case since μ_Y and σ_Y^2 are obtained from the transformed

data. What is needed then, is to solve for μ_y and σ_y^2 in terms of μ_x and σ_x^2. Substituting (20) into (21) results in:

$$\sigma_x^2 = \mu_x^2\left(e^{\sigma_y^2} - 1\right) \tag{22}$$

which when solved for $e^{\sigma_y^2}$, taking logs and simplifying, gives the solution of σ_y^2 as:

$$\sigma_y^2 = \ln\left(\frac{\sigma_x^2 + \mu_x^2}{\mu_x^2}\right). \tag{23}$$

Substituting (23) back into the log of (20), solving for μ_y and simplifying gives the solution of μ_y as:

$$\mu_y = \ln\left(\frac{\mu_x^2}{\sqrt{\mu_x^2 + \sigma_x^2}}\right). \tag{24}$$

Thus the mean and variance of the transformed data may be estimated from the sample mean and variance of the untransformed data reducing significantly the computational requirements.

APPENDIX E

DATA RECORD LAYOUT

5A Master Input File; 600 Characters Per Record, 10 Records Per Block, Fixed Length

CH POS	1	2-3	4-12	13-22	23-32	33-42	43-52	53	54
DATA	RECORD TYPE ("1")	COG	NIIN	REPLACE-MENT PRICE (B055)	COST TO REPAIR (B055A)	SET UP COST (B058)	REPAIR SET UP COST (B055A)	PROCUREMENT METHOD CODE (D025E)	NEWLY PROVISIONED INDICATOR (B067A)
PICTURE	1	2	9	8.2	8.2	8.2	8.2	1	1

CH POS	55	56	57	58	59	60
DATA	DEMAND INDICATOR (B067B)	HIGH DOLLAR DEMAND INDICATOR (B067C)	HIGH REPLACE-MENT PRICE INDICATOR (B067D)	PROGRAM RELATED INDICATOR (B067E)	REPAIRABLE ITEM INDICATOR (B067F)	AUTOMATIC REPAIR CYCLE INDICATOR (B067G)
PICTURE	1	1	1	1	1	1

CH POS	61	62-65	66-68	69-71	72-74	75-77	78-80
DATA	NAVY REPORTING REPAIRABLE INDICATOR (B067H)	PROCURE-MENT LEADTIME (B011A)	PROCURE-MENT LEAD-TIME M.A.D. (B011B)	REPAIR SURVIVAL RATE (F009)	REPAIR SURVIVAL RATE M.A.D. (F009A)	WEAR-OUT RATE (F007)	ITEM ESSEN-TIALITY (C008C)
PICTURE	1	2.2	2.1	1.2	1.2	1.2	.3

CH POS	81-83	84-86	87-96	97-106	107-116	117-126	127-136
DATA	SHELF LIFE (C028)	OBSO-LESCENCE RATE (B057)	SYSTEM DEMAND AVERAGE (B022)	SYSTEM DEMAND M.A.D. (A019)	SYSTEM OVER-HAUL DEMAND AVERAGE (B022A)	SYSTEM OVER-HAUL DEMAND M.A.D. (A019A)	SYSTEM CAR-CASS RETURN AVERAGE (B022B)
PICTURE	1.2	1.2	5.5	5.5	5.5	5.5	5.5

CH POS	137-146	147-152	153-157	158-167	168-177	178-185	186-193
DATA	SYSTEM CAR-CASS RETURN M.A.D. (A019B)	SYSTEM REQUISITION AVERAGE (A023B)	UNIT PACK (C021B)	ACQUISITION WAR RESERVE (B028C)	QUARTERLY DEMAND FORECAST (B074)	REPAIR LEVEL (B019B)	REPAIR QUANTITY (B021A)
PICTURE	5.5	4.2	5	10	8.2	8	8

CH POS	194-192	197-199	200-202	203-205	206-208
DATA	NUMBER POLICY RECEIVER ACTIVITIES	NAVY NONREPORTING/COMMERCIAL REPAIR TAT (B012)	NAVY NONREPORTING/COMMERCIAL REPAIR M.A.D. (B012B)	NAVY REPORTING REPAIR IN PROCESS TIME (B012C)	NAVY REPORTING REPAIR IN PROCESS TIME M.A.D. (B012D)
PICTURE	3	1.2	2.1	1.2	2.1

CH POS	209-212	213-222	223-232	233-302													
DATA	REPAIR PROBLEM TAT (B012E)	SYSTEM DEMAND END OF LEADTIME (B023D)	SYSTEM DEMAND DURING REPAIR TAT (B023H)	MAINTENANCE PROGRAM VALUES													
				P 1	P 2	P 3	P 4	P 5	P 6	P 7	P 8	P 9	P 10	P 11	P 12	P 13	P 14
PICTURE	2.2	8.2	8.2	5	5	5	5	5	5	5	5	5	5	5	5	5	5

CH POS	303-372														373-379	380-385	386
DATA	OVERHAUL PROGRAM VALUES														SYSTEM DUE IN (A008B)	SYSTEM BACKORDER QUANTITY (A011)	DOP INDICATOR
	P 1	P 2	P 3	P 4	P 5	P 6	P 7	P 8	P 9	P 10	P 11	P 12	P 13	P 14			
PICTURE	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	6	1

CH POS	387-390	391-397	398-404	405-411	412-418	419-425	426-432	433-439
DATA	MAXIMUM INDUCTION QUANTITY (B095)	ON-HAND WHOLESALE CONUS TIRS	ON-HAND NAS ALAMEDA	ON-HAND RETAIL CONUS TIRS	ON-HAND NAS CECIL FIELD	ON-HAND RETAIL OVERSEAS TIRS	NOT-FIT FOR ISSUE ON-HAND	M CONDITION ON-HAND
PICTURE	3	7	7	7	7	7	7	7

CH POS	440-446	447-453	454-460	461-467	468-474
DATA	TOTAL RESERVA-TIONS AT WHOLE-SALE CONUS TIRS (A013A)	ZG5 RESERVATIONS AT WHOLESALE CONUS TIRS (A013)	TOTAL RESERVATIONS AT NAS ALAMEDA (A013A)	ZG5 RESERVA-TIONS AT NAS ALAMEDA (A013)	TOTAL RESER-VATIONS AT RETAIL CONUS TIRS (A013A)
PICTURE	7	7	7	7	7

CH POS	475-481	482-488	489-495	496-502	503-509
DATA	ZG5 RESERVA-TIONS AT RE-TAIL CONUS TIRS (A013)	TOTAL RESERVA-TIONS AT NAS CECIL FIELD (A013A)	ZG5 RESERVA-TIONS AT NAS CECIL FIELD (A013)	TOTAL RESERVA-TIONS AT RETAIL OVERSEAS TIRS (A013A)	ZG5 RESERVATIONS-AT RETAIL OVER-SEAS TIRS (A013)
PICTURE	7	7	7	7	7

CH POS	510-512	513	514-520	521-527	528-534	535-544	545-554
DATA	NAS ALAMEDA ALLOCATION FACTOR	TYPE REPAIRABLE INDICATOR	PROTECTED MOBILIZA-TION REQ-UIREMENTS	POOL RESER-VATIONS	OUTFIT-TING RESER-VATIONS	ERROR TERM OF THE FIRST QUAR-TER - e1	ERROR TERM OF THE SECOND QUARTER e2
PICTURE	3	1	7	7	7	S5.5	S5.5

CH POS	555-564	565-574	575-585	586-595	596	597-600
DATA	EXPECTED ERROR VALUE - E(X)	VARIANCE - SIGMA SQUARED	BLANKS	STANDARD PRICE (B053)	BLANK	FAMILY GROUP CODE
PICTURE	S5.5	5.5	11	8.2	1	4

5A Demand Trailers; 600 Characters Per Record, 10 Records Per Block, Fixed Length

CH POS	1	2	3	4	5	6-9	10-13	14	15	16	17
DATA	RECORD TYPE ("2")	ENTRY POINT CODE	TYPE DEMAND CODE	RECURRING DEMAND CODE	PRIORITY	DAY	AMOUNT	BO LEVEL	ENTRY POINT CODE	TYPE DEMAND CODE	RECUR-RING DEMAND CODE
PICTURE	1	1	1	1	1	4	4	1	1	1	11

CH POS	18	19-20	23-26	27		587	588	589	590
DATA	PRIORITY	DAY	AMOUNT	BO LEVEL	.....	ENTRY POINT CODE	TYPE DEMAND CODE	RECUR-RING DEMAND CODE	PRIORITY
PICTURE	1	4	4	1		1	1	1	1

CH POS	591-594	595-598	599	600
DATA	DAY	AMOUNT	BO LEVEL	"0"
PICTURE	4	4	1	1

APPENDIX F

FORTRAN PROGRAMS

```
//YOUNT$$A JOB (1642,0045),'MARK YOUNT',CLASS=B,MSGCLASS=Z
//  EXEC FORTXCLG
//FORT.SYSIN DD *
C***********************************************************************
C**                                                                   **
C**   MVSTOVM PROGRAM READS THE RAW DATA FROM THE MASS STORAGE        **
C**   DEVICE AND COPIES THE INPUT RECORDS TO A TRANSFER DISK THAT     **
C**   IS ACCESSIBLE TO THE VM SYSTEM.  THE MAIN DATA RECORDS AND      **
C**   SUBRECORDS ARE TREATED DIFFERENTLY IN THAT FOR THE MAIN         **
C**   RECORDS, ONLY THE RECORD/SUBRECORD INDICATOR IS COPIED.  THE    **
C**   REMAINING DATA ELEMENTS REPRESENTING INFORMATION NOT USED IN    **
C**   THIS ANALYSIS ARE DISCARDED.  ALL DEMAND SUBRECORDS FOR EACH    **
C**   MAIN RECORD ARE COPIED UNALTERED.  BY SETTING THE VALUES OF     **
C**   THE VARIABLES START AND FREQ, THE USER IS ABLE TO SELECT        **
C**   ONLY A PORTION OF THE RAW DATA FOR TRANSFER.  IN THIS WAY,      **
C**   INDEPENDENT SETS OF DATA COULD BE EVALUATED TO VERIFY THE       **
C**   POWER OF THE PROCEDURE.                                         **
C**                                                                   **
C**   THIS PROGRAM IS DESIGNED TO TAKE A 1 IN FREQ SAMPLE FROM THE    **
C**   FMSO 5A SIMULATOR ASO DEMAND DATA STORED ON                     **
C**   MSS.S1642.THESIS1 FOR 1R DATA AND MSS.S1642.THESIS2 FOR 2R      **
C**   DATA.  THE FILEDEF FOR FT01F001 MUST BE CHANGED FOR             **
C**   WHICHEVER DATA SET IS REQUIRED.  USE MVSGET EXEC TO RETRIEVE    **
C**   THE SAMPLE FROM MVS004.                                         **
C**                                                                   **
C**   DEFINITION OF VARIABLES:                                        **
C**                                                                   **
C**      A(599)    -- CHARACTER ARRAY USED TO TRANSFER DATA           **
```

```
C**       EOF        -- WRITTEN TO OUTPUT FILE AS END OF FILE INDICATOR **
C**       FREQ       -- SAMPLING FREQUENCY                               **
C**       IN         -- COUNT OF RECORDS READ                            **
C**       N          -- 1 = RECORD/2 = SUBRECORD INDICATOR               **
C**       OUT        -- COUNT OF RECORDS WRITTEN                         **
C**       START      -- RECORD NUMBER OF FIRST RECORD TO READ            **
C**                                                                      **
C** NOTE:  TO COPY ENTIRE FILE TO MVS004, SET START = 0 AND FREQ = 1    **
C**                                                                      **
C************************************************************************
      INTEGER*4
     *    A(599),         N,              FREQ,           START,
     *    EOF/0/,         IN/0/,          OUT/0/
C
C     SET FREQ TO DESIRED SAMPLING FREQUENCY.
C
      FREQ = 1
C
C     SET START TO DESIRED FIRST LINE ITEM.
C
      START = 0
      READ(1,800,END=80) N
 10   CONTINUE
      IF (.NOT.((N.EQ.1).AND.(MOD(IN+1,FREQ).EQ.START))) GOTO 40
C
C        READ AND TRANSFER TO MVS004 DESIRED RECORDS AND SUBRECORDS
C
         IN = IN + 1
         OUT = OUT + 1
         WRITE(2,800) N
         READ(1,810,END=80) N,(A(I),I=1,599)
 20      CONTINUE
         IF (.NOT.(N .EQ. 2)) GOTO 30
              WRITE(2,810) N,(A(I),I=1,599)
              READ(1,810,END=80) N,(A(I),I=1,599)
         GOTO 20
```

```
30          CONTINUE
      GOTO 70
40    CONTINUE
C
C           SKIP BY UNWANTED RECORDS AND SUBRECORDS
C
            IN = IN + 1
            READ(1,800,END=80) N
50          CONTINUE
            IF (.NOT.(N .EQ. 2)) GOTO 60
                 READ(1,800,END=80) N
            GOTO 50
60          CONTINUE
70    CONTINUE
      GOTO 10
80    CONTINUE
C
C     WRITE END OF FILE MARKER
C
      WRITE(2,800) EOF
      FREQ = FREQ - 1
      WRITE(2,820) START,FREQ,IN,OUT
      STOP
800   FORMAT(I1)
810   FORMAT(I1,8(75A1))
820   FORMAT(/'STARTING AT RECORD ',I3,' AND SKIPPING EVERY ',I3,
     *    ' RECORDS',/,I5,' RECORDS WERE READ IN AND',I5,' RECORDS ',
     *    'WERE OUTPUT.')
      END
/*
//GO.FT01F001 DD DISP=SHR,DSN=MSS.S1642.THESIS1
//GO.FT02F001 DD UNIT=3350,VOL=SER=MVS004,DISP=(NEW,KEEP),
//    DCB=(RECFM=FB,LRECL=600,BLKSIZE=6000),SPACE=(CYL,(4,1)),
//    DSN=S1642.OUTPUT
//
```

```
C*****************************************************************
C**                                                             **
C**   THESIS IS THE MAIN PROGRAM FOR THE RESAMPLING PROCEDURE.  **
C**   THAT FUNCTIONS AS A CONTROL OVER THE DATA                 **
C**   BEING EVALUATED BY CHECKING PARAMETERS RETURNED BY        **
C**   SUBROUTINES.  IF THE PARAMETERS ARE OUTSIDE ACCEPTABLE    **
C**   LIMITS, THE ENTIRE SAMPLE IS DISCARDED BEFORE FURTHER     **
C**   PROCESSING.  THE PARAMETERS CHECKED ARE:                  **
C**       1.  NUMBER OF NON-ZERO OBSERVATIONS GREATER THAN ONE, **
C**       2.  STANDARD DEVIATION OF ANY DEMAND SERIES EQUALS ZERO. **
C**   THE SAMPLES ARE ALSO CATEGORIZED INTO LOW, MEDIUM AND HIGH **
C**   DEMAND CATEGORIES AND COUNTS KEPT FOR THE NUMBER IN EACH  **
C**   CATEGORY.                                                 **
C**                                                             **
C**   DEFINITIONS OF VARIABLES:                                 **
C**                                                             **
C**      AN       -- FLOAT OF NBR                               **
C**      CAT      -- BREAKPOINTS OF DEMAND CATEGORIES FOR LOW,  **
C**                  MEDIUM AND HIGH DEMANDS                    **
C**      DY       -- NUMERIC CALENDAR DAY OF MONTH              **
C**      ERROR    -- LOGICAL TEST FOR ERRORS IN STATISTICAL VALUES **
C**      FIRST    -- LOGICAL VARIABLE TRUE ONLY FOR FIRST SAMPLE **
C**      FX       -- DISTRIBUTION PERCENTILES TO TEST           **
C**      HR       -- HOUR                                       **
C**      I        -- DEMAND CATEGORY FOR SAMPLE UNDER TEST      **
C**      IC       -- CCUNT OF SAMPLES IN EACH DEMAND CATEGORY   **
C**      ICNOUT   -- DSRN FOR TERMINAL OUTPUT                   **
C**      IDSK2    -- DSRN OF PROGRAM CONTROL PARAMETERS         **
C**      IER      -- RETURN CODE FROM STATS SUBROUTINE          **
C**      IPRINT   -- DSRN OF PRINTER DISK                       **
C**      IPRT     -- CONTROL PARAMETER FOR PRINTING DIAGNOSTICS **
C**      ISEED    -- SEED FOR RANDOM NUMBER GENERATOR USED IN SAMPLE **
C**      J        -- DO INDEX FOR GENERAL USE                   **
C**      MI       -- MINUTES                                    **
C**      MO       -- NUMERIC MONTH OF YEAR                      **
C**      MODELS   -- NUMBER OF MODELS TO STUDY                  **
```

```
C**      MONTH     -- CHARACTER MONTHS OF THE YEAR                      **
C**      MSE       -- MEAN SQUARED ERROR OF PREDICTED VS. ACTUAL        **
C**                   PERCENTILE                                        **
C**      MU        -- SAMPLE MEAN OF EACH GROUP OF OBSERVATIONS         **
C**      NBR       -- NUMBER OF DEMAND OBSERVATIONS TO TEST IS 48       **
C**      NBROBS    -- NUMBER OF DEMAND OBSERVATIONS IN EACH GROUP,      **
C**                   NBROBS(1) ALWAYS EQUALS NBR                       **
C**      NPTILE    -- NUMBER OF SAMPLE PERCENTILES TO TEST              **
C**      NRECD     -- MAXIMUM NUMBER OF LINE ITEMS TO PROCESS           **
C**      NREPS     -- NUMBER OF PSEUDO SAMPLE REPETITIONS               **
C**      OBS       -- ARRAY OF DEMAND OBSERVATIONS WHERE COLUMNS ARE    **
C**                   1.  RAW OBSERVATIONS AFTER REMOVING NEGATIVES     **
C**                   2.  NON-ZERO DEMAND OBSERVATIONS                  **
C**                   3.  LOG OF NON-ZERO DEMAND OBSERVATIONS           **
C**      P         -- PROBABILITY OF A NON-ZERO DEMAND                  **
C**      PHAT      -- COUNTER FOR ACCUMULATING PERCENTILE ESTIMATES     **
C**      SC        -- SECONDS                                           **
C**      STDV      -- SAMPLE STANDARD DEVIATION OF EACH DEMAND GROUP    **
C**      X         -- ABSCISSA VALUE OF EACH PROBABILITY MODEL UNDER    **
C**                   TEST DETERMINED BY SUBROUTINE XINV                **
C**      YR        -- TWO DIGIT VALUE OF YEAR                           **
C**                                                                     **
C**    SUBROUTINES REQUIRED:                                            **
C**                                                                     **
C**      OPEN      -- SETS UP INITIAL FILE DEFINITIONS                  **
C**      INITAL    -- READS RAW DATA FROM DISK AND CREATS AN ARRAY      **
C**                   OF THREE DEMAND OBSERVATIONS AS FOLLOWS:          **
C**                   1.  RAW DATA                                      **
C**                   2.  NON-ZERO OBSERVATIONS ONLY                    **
C**                   3.  LOG OF NON-ZERO OBSERVATIONS                  **
C**      TSTATS    -- COMPUTES MEAN AND STANDARD DEVIATIONS OF ABOVE    **
C**      XINV      -- GIVEN A PROBABILITY, COMPUTES THE ABSCISSA X      **
C**                   VALUE FROM THE DISTRIBUTION INVERSE FUNCTION      **
C**      SAMPLE    -- GENERATES PSEUDO SAMPLES AND TESTS AGAINST        **
C**                   THE X VALUE COMPUTED IN XINV.  ACCUMULATES        **
C**                   BERNOULLI SUCCESS AND AVERAGE MEAN SQUARE ERROR   **
```

```
C**       DATIME     -- GETS TIME AND MONTH,DAY AND YEAR FROM SYSTEM     **
C**       OUTPUT     -- GENERATES OUTPUT REPORT                          **
C**                                                                      **
C************************************************************************
      INTEGER*4
     *   YR,             MO,              DY,              JD,
     *   HR,             MI,              SC,              HD,
     *   IDSK,           I,               IDSK2,           IER,
     *   IPRINT,         IPRT,            NBR/48/,         NRECD,
     *   NPTILE/5/,      NREPS,          .IC(3)/3*0/,      NBROBS(3),
     *   ISEED/392746/,  MODELS/10/,      J,               NBRDEL/0/,
     *   ICNOUT,         NBRNEG/0/,       MONTH(12)/'JAN','FEB','MAR',
     *   'APR','MAY','JUN','JUL','AUG','SEP','OCT','NOV','DEC'/
      REAL*4
     *   AN,                              P,
     *   MU(3),                           STDV(3),
     *   X(10,5),                         OBS(48,3),
     *   MSE(3,10,5)/150*0.0/,            PHAT(3,10,5)/150*0.0/,
     *   CAT(4)/0.0,1.0,20.0,9999.99/, FX(5)/0.75,0.80,0.85,0.90,0.95/
      LOGICAL*1
     *   ERROR,          FIRST/.TRUE./
      COMMON /FILES/
     *   IPRT,           ICONIN,          ICNOUT,          IPRINT,
     *   IDSK(30)
      CALL OPEN('THESIS')
      IDSK2 = IDSK(2)
      READ(IDSK2,*) NRECD,NREPS
      CAT(2) = CAT(2) / 12.0
      CAT(3) = CAT(3) / 12.0
      AN = FLOAT(NBR)
10    CONTINUE
      CALL INITAL(FIRST,NBR,OBS,NBROBS,NBRNEG,&80)
      FIRST = .FALSE.
      P = FLOAT(NBROBS(2)) / AN
C
C     RETURN BASIC STATISTICS FOR EACH GROUP OF OBSERVATIONS
```

```
C
      ERROR = .FALSE.
      DO 20 J=1,3
         CALL TSTATS(OBS(1,J),NBROBS(J),MU(J),STDV(J),IER)
         IF (IER .NE. 0) ERROR = .TRUE.
20    CONTINUE
C
C     SKIP ENTIRE SAMPLE IF TOO SMALL OR STANDARD DEVIATION EQUALS 0
C
      IF (ERROR) NBRDEL = NBRDEL + 1
      IF (ERROR) GOTO 70
      IF (IPRT .GE. 5) WRITE(IPRINT,600) MU(1),STDV(1)
      IF (IPRT .GE. 5) WRITE(IPRINT,610) MU(2),STDV(2),P
      IF (IPRT .GE. 5) WRITE(IPRINT,620) MU(3),STDV(3),NBROBS(3)
C
C     DIVIDE INTO DEMAND CATEGORIES BY OVERALL MEAN
C
      IF (MU(1) .GT. CAT(2)) GOTO 30
         I = 1
      GOTO 50
30    CONTINUE
      IF (MU(1) .GT. CAT(3)) GOTO 40
         I = 2
      GOTO 50
40    CONTINUE
         I = 3
50    CONTINUE
      IC(I) = IC(I) + 1
C
C     COMPUTE LOCATION OF EACH PERCENTILE FOR ALL MODELS.
C
      CALL XINV(P,MU,STDV,FX,MODELS,NPTILE,X,IER)
C
C     CONDUCT RESAMPLING PROCEDURE RETURNING THE NUMBER OF
C     BERNOULLI SUCCESSES AND THE AVERAGE MEAN SQUARED ERROR OF THE
C     MODEL.
```

```
C
      DO 60 J=1,NREPS
         CALL SAMPLE(ISEED,OBS,NBR,I,MODELS,NPTILE,X,FX,PHAT,MSE)
60    CONTINUE
70    CONTINUE
      IF (IC(1)+IC(2)+IC(3) .LT. NRECD) GOTO 10
80    CONTINUE
C
C     GENERATE OUTPUT REPORT
C
      CALL DATIME(YR,MO,DY,JD,HR,MI,SC,HD)
      WRITE(ICNOUT,630) NBRDEL,NBRNEG,(IC(I),I=1,3)
      WRITE(IPRINT,630) NBRDEL,NBRNEG,(IC(I),I=1,3)
      WRITE(ICNOUT,640) HR,MI,SC,DY,MONTH(MO),YR
      WRITE(IPRINT,640) HR,MI,SC,DY,MONTH(MO),YR
      CALL OUTPUT(MODELS,NPTILE,NREPS,FX,IC,PHAT,MSE,CAT)
      STOP
600   FORMAT('0SAMPLE MEAN: ',F10.4,'  SAMPLE STANDARD DEVIATION:',
     *   F10.4)
610   FORMAT('0SAMPLE MEAN: ',F10.4,'  SAMPLE STANDARD DEVIATION:',
     *   F10.4,'  PROBABILITY OF A NON-ZERO OBSERVATION: ',F10.8)
620   FORMAT('0SAMPLE MEAN: ',F10.4,'  SAMPLE STANDARD DEVIATION:',
     *   F10.4,'  NUMBER OF NON-ZERO OBSERVATIONS:        ',I10)
630   FORMAT('1',20X,'SUMMARY RUN STATISTICS',/,
     *       '-NUMBER OF SAMPLES DELETED BY POOR STATISTICS: ',I3,/,
     *       '0NUMBER OF SAMPLES WITH NEGATIVE DEMANDS:      ',I3,/,
     *       '0PROCESSED ',I4,' RECORDS ASSIGNED AS LOW DEMAND'/,
     *       '0PROCESSED ',I4,' RECORDS ASSIGNED AS MEDIUM DEMAND'/,
     *       '0PROCESSED ',I4,' RECORDS ASSIGNED AS HIGH DEMAND')
640   FORMAT('0REPORT GENERATED AT ',I2,':',I2,':',I2,' ON ',
     *   I2,' ',A3,' 19',I2)
      END
```

```
      SUBROUTINE OPEN (FNAME$)
C*********************************************************************
C**                                                                 **
C**   OPEN IS A GENERAL PURPOSE SUBROUTINE DESIGNED TO HANDLE ALL   **
C**   THE OPERATING SUSTEM FILE DEFINITIONS REQUIRED TO INPUT AND   **
C**   OUTPUT DATA TO THE REQUIRED DEVICE.  OPEN IS CALLED AS THE    **
C**   FIRST EXECUTABLE STATEMENT OF THE MAIN PROGRAM AND READS A    **
C**   DATA FILE TO DETERMINE THE DATA SET REFERENCE NUMBERS (DSRN)  **
C**   REQUIRED BY THE MAIN PROGRAM.  OPEN THEN CALLS THE SYSTEM     **
C**   SUBROUTINE FRTCMS TO EXECUTE THE CMS FILEDEF COMMANDS WITH    **
C**   ARGUMENTS SPECIFIED IN THE DATA FILE.  OPEN ALSO INITIALIZES  **
C**   A COMMON BLOCK WITH THE DSRN'S ASSIGNED TO MAKE THEM          **
C**   ACCESSIBLE TO OTHER PROGRAM UNITS.                            **
C**   TO OPERATE, THE USER MUST HAVE A FILE NAMED 'FNAME$           **
C**   FILES'.  OPEN READS THE FILE AS FOLLOWS:                      **
C**                                                                 **
C**      LINE 1.   FORMAT(4I5,5X,A8) ASSIGNED TO OUTPUT CONTROL     **
C**                VARIABLE IPRT AND I/O DEVICE NUMBERS FOR         **
C**                TERMINAL INPUT, TERMINAL OUTPUT, PRINTER DISK,   **
C**                AND FILENAME FOR PRINT FILE '<FN> OUTPUT A1'     **
C**      LINE 2.   FORMAT(I5,5X,7(A8,2X)) ASSIGNED TO I/O DEVICE    **
C**                NUMBERS FOR OTHER THAN PRINTER DISK I/O AND THE  **
C**                FILENAME AND FILETYPE.  FILEMODE DEFAULTS TO A1. **
C**                REMAINING VALUES ARE FIVE OR LESS OPTIONAL       **
C**                FILEDEF PARAMETERS, IE. LRECL, BLOCK, ETC.       **
C**                                                                 **
C**      LINE 2 IS OPTIONAL AND MAY BE LEFT OUT OR REPEATED UP TO   **
C**      THIRTY TIMES AS NECESSARY.                                 **
C**                                                                 **
C**      OTHER SUBROUTINES CALLED:  STR$                            **
C**                                                                 **
C**   WRITTEN BY MARK YOUNT            AUG 1981                     **
C**                                                                 **
C*********************************************************************
C
      IMPLICIT REAL*8 (A-H,O-Z)
```

```
      COMMON /FILES/ IPRT,ICONIN,ICNOUT,IPRINT,IDSK(30)
      REAL*8 PAR(14)
     *  /'FILEDEF ',     'TERMINAL',     'DISK    ',     'OUTPUT  ',
     *   'A1      ',     '(       ',     'RECFM   ',     'FA      ',
     *   'BLOCK   ',     '133     ',     'PERM    ',     'FILES   ',
     *   ')       ',     '        '/
      DIMENSION OPT$(5)
C
      IPRT = 0
      DNUM$ = STR$(99)
      CALL FRTCMS(PAR(1),DNUM$,PAR(3),FNAME$,PAR(12),PAR(5),
     *   PAR(6),PAR(11))
C
C     READ DATA FOR TERMINAL I/O AND PRINTER DESTINED OUTPUT
C
      READ(99,100) ITMP,ICONIN,ICNOUT,IPRINT,PNAME$
100   FORMAT(4I5,5X,A8)
      CONIN$ = STR$(ICONIN)
      CALL FRTCMS(PAR(1),CONIN$,PAR(2),PAR(6),PAR(11))
      CNOUT$ = STR$(ICNOUT)
      CALL FRTCMS(PAR(1),CNOUT$,PAR(2),PAR(6),PAR(11))
      PRINT$ = STR$(IPRINT)
      IF (IPRINT .EQ. 6) GOTO 10
      CALL FRTCMS(PAR(1),PRINT$,PAR(3),PNAME$,PAR(4),PAR(5),PAR(6),
     *   PAR(7),PAR(8),PAR(9),PAR(10),PAR(11))
10    CONTINUE
C
C     READ DATA FOR DISK FILE OPERATIONS
C
110   FORMAT(I5,5X,7(A8,2X))
      IPRT = ITMP
      DO 20 I=1,30
         READ(99,110,END=30) IDSK(I),DNAME$,DTYPE$,(OPT$(J),J=1,5)
         DO 15 J=1,5
              IF (OPT$(J) .NE. PAR(14)) GOTO 15
              OPT$(J) = PAR(13)
```

```
              GOTO 16
15         CONTINUE
16         CONTINUE
           DNUM$ = STR$(IDSK(I))
           IF(IPRT .GE. 20) WRITE(ICNOUT,200) PAR(1),DNUM$,PAR(3),DNAME$,
     *        DTYPE$,PAR(6),PAR(11),(OPT$(J),J=1,5)
200        FORMAT(' ',7A10/,10X,5A10)
           CALL FRTCMS(PAR(1),DNUM$,PAR(3),DNAME$,DTYPE$,PAR(6),PAR(11),
     *        OPT$(1),OPT$(2),OPT$(3),OPT$(4),OPT$(5))
20       CONTINUE
30       CONTINUE
         IF (IPRT .GE. 10) CALL FRTCMS(PAR(1))
         REWIND 99
         RETURN
         END
```

```
      DOUBLE PRECISION FUNCTION STR$(IVAL)
C***********************************************************************
C**                                                                   **
C**   STR$ IS A GENERAL PURPOSE FUNCTION INVOKED BY OPEN DESIGNED     **
C**   TO ENHANCE THE FLEXIBILITY OF THE FRTCMS SUBROUTINE BY          **
C**   CONVERTING INTEGER DSRN'S INTO REAL*8 CHARACTER STRINGS         **
C**   SUITABLE AS ARGUMENTS FOR FRTCMS.  (FRTCMS USES ONLY            **
C**   CHARACTER STRING ARGUMENTS, ALPHABETIC ARGUMENTS MAY BE FROM    **
C**   1 TO 8 CHARACTERS LEFT JUSTIFIED, BUT NUMERIC ARGUMENTS MUST    **
C**   BE EIGHT CHARACTERS LEFT JUSTIFIED AND PADDED ON THE RIGHT      **
C**   WITH BLANKS IF NECESSARY.)                                      **
C**   STR$ CONVERTS INTEGER VALUES IN THE RANGE OF +99,999,999 TO     **
C**   -9,999,999 INTO ALPHANUMERIC REPRESENTATION AS THE FUNCTION     **
C**   OUTPUT.  STR$ MUST BE TYPED REAL*8 IN THE CALLING PROGRAM.      **
C**                                                                   **
C**   STR$ RETURNS A CHARACTER VALUE OF 0 IF CONVERSION UNSUCCESSFUL. **
C**                                                                   **
C**   WRITTEN BY MARK YOUNT              SEP 1981                     **
C**                                                                   **
C***********************************************************************
      REAL*8 B$,BLNK/'        '/
      COMMON /FILES/ IPRT,ICONIN,ICNOUT,IPRINT,IDSK(30)
      LOGICAL*1 NUM(8),ONUM(8),MINUS/'-'/
      LOGICAL*1 DIGIT(10)/'0','1','2','3','4','5','6','7','8','9'/
      INTEGER*4 FUNC$/'STR$'/
      EQUIVALENCE (ONUM(1),B$)
C
      IF (IPRT .GE. 20) WRITE(ICNOUT,201) IVAL
201   FORMAT(' INPUT INTEGER = ',I10)
      IM = 8
      B$ = BLNK
      IF ((IVAL .GE. 10**8) .OR. (IVAL .LE. -1*10**7)) GOTO 6000
      ITEMP = IABS(IVAL)
C
C     EXTRACT CHARACTERS ONE AT A TIME FROM IVAL AND STORE AS
C     ALPHANUMERIC CHARACTERS IN ARRAY NUM
```

```
C
      DO 10 I=1,8
         IF (ITEMP .EQ. 0) GOTO 20
              N = ITEMP
              ITEMP = ITEMP / 10
              N = N - ITEMP * 10
              NUM(IM) = DIGIT(N+1)
              IF (IPRT .GE. 40) WRITE(ICNOUT,200) N,NUM(IM)
200           FORMAT(' CONVERTED NUMERIC ',I1,' TO CHARACTER ',A1)
              IM = IM - 1
10    CONTINUE
20    CONTINUE
C
C     LEFT JUSTIFY CHARACTERS IN B$(1)
C
      DO 60 I=1,8
         IF (I .GT. 1) GOTO 50
              IF (IVAL) 30,40,50
30            CONTINUE
                   ONUM(I) = MINUS
                   IM = IM - 1
                   IF (IPRT .GE. 40) WRITE(ICNOUT,210)
210                FORMAT(' ASSIGNED NEGATIVE IVALUE TO STR$')
                   GOTO 60
40            CONTINUE
                   ONUM(I) = DIGIT(1)
                   IM = IM - 1
                   IF (IPRT .GE. 40) WRITE(ICNOUT,200) IVAL,ONUM(I)
                   GOTO 60
50       CONTINUE
         J = I + IM
         IF (J .GT. 8) GOTO 60
         ONUM(I) = NUM(J)
60    CONTINUE
C
C     CREATE OUTPUT ARRAY
```

```
C
      STR$ = B$
      IF (IPRT .GE. 30)  WRITE(ICNOUT,220) IVAL,STR$
220   FORMAT(' NUMERIC VALUE = ',I8,' ALPHANUMERIC VALUE = ',A8)
      RETURN
C
C     ERROR HANDLING SECTION FOLLOWS
C
6000  CONTINUE
      WRITE(ICNOUT,6010) FUNC$
6010  FORMAT(' *** STRING LENGTH ERROR *** ',A4)
      STR$ = DIGIT(1)
      RETURN
      END
```

```
      SUBROUTINE INITAL(FIRST,M,OBS,NBROBS,NBRNEG,*)
C*************************************************************************
C**                                                                     **
C**   INITAL FIRST CALLS SUBROUTINE DATARD TO READ ONE SAMPLE           **
C**   FROM THE SPECIFIED DISK WHEN CALLED FROM THE MAIN PROGRAM.        **
C**   THE STRING OF FORTY-EIGHT DEMAND OBSERVATIONS THUS READ IN        **
C**   ARE FIRST PROCESSED TO REMOVE ANY NEGATIVE DEMAND                 **
C**   OBSERVATIONS BY BACKING OUT THE PRIOR POSITIVE DEMAND THAT        **
C**   IS EQUAL OR LARGER THAN THE NEGATIVE VALUE.  WHETHER A            **
C**   LARGER VALUE IS FOUND OR NOT, THE NEGATIVE DEMAND IS SET TO       **
C**   ZERO.  THE RESULTING DEMAND SERIES IS THE EDITED DEMAND           **
C**   SERIES USED FOR FURTHER PROCESSING.  TWO MORE STEPS ARE           **
C**   REQUIRED PRIOR TO SUBROUTINE RETURN, FIRST A SECOND DEMAND        **
C**   SERIES IS CONSTRUCTED OF ONLY THE NON-ZERO DEMANDS AND THE        **
C**   COUNT STORED, SECOND A THIRD DEMAND SERIES IS CONSTRUCTED OF      **
C**   THE NATURAL LOG OF THE NON-ZERO DEMAND SERIES JUST CREATED.       **
C**   THUS WHEN INITAL RETURNS CONTROL TO THE MAIN PROGRAM, THREE       **
C**   DEMAND SERIES HAVE BEEN CONSTRUCTED                               **
C**                                                                     **
C**   DEFINITION OF VARIABLES:                                          **
C**                                                                     **
C**      COUNT     -- TEMPORARY COUNTER EVENTUALLY ASSIGNED TO          **
C**                   NBROBS(2) AND NBROBS(3)                           **
C**      FIRST     -- LOGICAL VARIABLE TO INDICATE FIRST USE OF INITAL**
C**      I,J,K     -- DO LOOP INDICIES                                  **
C**      IDSK1     -- INPUT DSRN FOR DEMAND DATA                        **
C**      IPRINT    -- OUTPUT DSRN FOR LOCAL PRINTER                     **
C**      IPRT      -- CONTROLS LEVEL OF DIAGNOSTICS PRINTED             **
C**      IREV      -- METHOD TO COUNT BACKWARD IN OBS ARRAY             **
C**      M         -- INPUT COUNT OF OBSERVATIONS TO INPUT,             **
C**                   NBROBS(1) ALWAYS EQUALS M                         **
C**      NBRNEQ    -- COUNTS NUMBER OF SAMPLES WITH NEGATIVE OBS.       **
C**      NBROBS    -- COUNTS OF OBSERVATIONS IN EACH OF ABOVE           **
C**      NEGOBS    -- LOGICAL VARIABLE, TRUE IF SAMPLE HAS NEG OBS.     **
C**      OBS(M,1)  -- ORIGINAL OBSERVATIONS WITH NEGATIVES REMOVED      **
C**      OBS(M,2)  -- ALL ZERO OBSERVATIONS REMOVED FROM DATA           **
```

```
C**       OBS(M,3)   -- NATURAL LOG OF NON-ZERO OBSERVATIONS                **
C**                                                                         **
C**    OTHER SUBROUTINES REQUIRED BY INITAL:                                **
C**                                                                         **
C**       DATARD     -- DOES ACTUAL READING OF DATA FROM RAW DATA SETS      **
C**                                                                         **
C****************************************************************************
      INTEGER*4
     *   IPRT,              ICONIN,            ICNOUT,            IPRINT,
     *   IDSK(30),          IREV,              IDSK1,             NBROBS(3),
     *   COUNT,             I,                 J,                 K,
     *   M,                 NBRNEG
      REAL*4
     *   OBS(M,3)
      LOGICAL*1
     *   NEGOBS,            FIRST
      COMMON /FILES/
     *   IPRT,              ICONIN,            ICNOUT,            IPRINT,
     *   IDSK
      IDSK1 = IDSK(1)
      NBROBS(1) = M
      CALL DATARD(OBS(1,1),M,FIRST,&40)
C
C     ECHO PRINT INPUT VALUES (CONTROLLED BY IPRT VALUE)
C
      IF (IPRT .GE. 5) WRITE(IPRINT,600) (OBS(K,1),K=1,M)
      NEGOBS = .FALSE.
      DO 20 I=1,M
         IF (OBS(I,1) .GE. 0.0) GOTO 20
C
C        EDIT BY BACKING OUT AND REMOVING NEGATIVE OBSERVATIONS
C
         NEGOBS = .TRUE.
         DO 10 J = 2,I
            IREV = I - J + 1
            IF (OBS(IREV,1) .LT. -OBS(I,1)) GOTO 10
```

```
                  OBS(IREV,1) = OBS(IREV,1) + OBS(I,1)
                  OBS(I,1) = 0.0
                  GOTO 20
10          CONTINUE
            OBS(I,1) = 0.0
20       CONTINUE
         IF (NEGOBS) NBRNEG = NBRNEG + 1
C
C        PRINT EDITED INPUT VALUES (CONTROLLED BY IPRT)
C
         IF (IPRT .GE. 5) WRITE(IPRINT,610) (OBS(K,1),K=1,M)
C
C        CREATE ARRAY OF ONLY NON-ZERO OBSERVATIONS
C        AND ARRAY OF LOG OF THESE NON-ZERO OBSERVATIONS
C
         COUNT = 0
         DO 30 I=1,M
            IF (OBS(I,1) .EQ. 0.0) GOTO 30
               COUNT = COUNT + 1
               OBS(COUNT,2) = OBS(I,1)
               OBS(COUNT,3) = LOG(OBS(I,1))
30       CONTINUE
         NBROBS(2) = COUNT
         NBROBS(3) = COUNT
C
C        PRINT NON-ZERO INPUT VALUES (CONTROLLED BY IPRT)
C
         IF (IPRT .GE. 5) WRITE(IPRINT,620) (OBS(K,2),K=1,COUNT)
C
C        PRINT LOG OF NON-ZERO INPUT VALUES (CONTROLLED BY IPRT)
C
         IF (IPRT .GE. 5) WRITE(IPRINT,630) (OBS(K,3),K=1,COUNT)
         RETURN
40       CONTINUE
         RETURN 1
600      FORMAT('1RAW OBSERVATIONS ',3(T20,20F5.0,/))
```

```
610    FORMAT('ONEGATIVE REMOVED ',3(T20,20F5.0,/))
620    FORMAT('ONON ZERO OBS ONLY',3(T20,20F5.0,/))
630    FORMAT('OLOG NON-ZERO ONLY',3(T20,20F5.0,/))
800    FORMAT(50F4.0)
       END
```

```
      SUBROUTINE DATARD(MONTH,NBR,TIME,*)
C*********************************************************************
C**                                                                 **
C**   DATARD DOES THE ACTUAL READING OF DATA FROM THE VM MINIDISK   **
C**   AS TWO PARALLEL ARRAYS OF DATA, THE FIRST THE SEQUENTIAL DAY  **
C**   OF DEMAND AND THE SECOND THE QUANTITY DEMANDED.  ALL THE      **
C**   INDIVIDUAL DEMANDS FOR EACH DAY ARE TOTALED AND THEN GROUPED  **
C**   INTO THIRTY DAY BUCKETS REPRESENTING THE ACCUMULATED DEMAND   **
C**   FOR THIS SAMPLE FOR ONE MONTH.  THE DATA RETURNED FROM        **
C**   DATARD IS THE RAW MONTHLY DEMAND DATA TO BE PROCESSED BY      **
C**   SUBROUTINE INITAL.                                            **
C**                                                                 **
C**   DEFINITION OF VARIABLES:                                      **
C**                                                                 **
C**      DATE     -- SEQUENTIAL DATE OF DEMAND, DAY 1 TO DAY 1440   **
C**                  ONE ELEMENT FOR EACH ELEMENT OF INPUT DATA     **
C**      DEM      -- ACCUMULATES DAILY DEMANDS FOR LATER AGGREGATION **
C**      I,J      -- DO LOOP INDICIES                               **
C**      IDSK1    -- INPUT DSRN FOR DEMAND DATA                     **
C**      FIRST    -- STARTING INDEX OF CURRENT 30 DAY MONTH BLOCK   **
C**      LAST     -- ENDING INDEX OF CURRENT 30 DAY MONTH BLOCK     **
C**      MONTH    -- OUTPUT DEMAND VALUES AGGRAGATED IN 30 DAY BLOCKS**
C**      NBR      -- NUMBER OF OBSERVATIONS TO CREATE               **
C**      QUANT    -- QUANTITY DEMANDED ON SEQUENTIAL DATE           **
C**      TIME     -- LOGICAL VARIABLE TRUE ONLY ON FIRST CALL       **
C**                                                                 **
C*********************************************************************
      INTEGER*4
     *   IDSK,         FIRST,        LAST,         IDSK1,
     *   DATE(46),     I,            J,            N
      REAL*4
     *   DEM(1550),    MONTH(NBR),   QUANT(46)
      LOGICAL*1
     *   TIME
      COMMON /FILES/
     *   IPRT,         ICONIN,       ICNOUT,       IPRINT,
```

```
     *    IDSK(30)
      IDSK1 = IDSK(1)
      IF (TIME) READ(IDSK1,800) N
C
C     INITIALIZE COUNTERS USED TO TOTAL DEMAND
C
      DO 20 I=1,1550
           DEM(I) = 0.0
20    CONTINUE
      DO 30 I=1,NBR
           MONTH(I) = 0.0
30    CONTINUE
C
C     READ IN DEMANDS AND ACCUMULATE IN DAILY BUCKETS
C
      READ(1,810) N,(DATE(I),QUANT(I),I=1,46)
40    CONTINUE
      IF (.NOT.(N .EQ. 2)) GOTO 70
         DO 60 I=1,46
              IF (.NOT.(DATE(I) .NE. 0)) GOTO 50
                   DEM(DATE(I)) = DEM(DATE(I)) + QUANT(I)
50            CONTINUE
60       CONTINUE
         READ(1,810) N,(DATE(I),QUANT(I),I=1,46)
      GOTO 40
70    CONTINUE
C
C     AGGREGATE ACCUMULATED DEMANDS INTO MONTHLY BUCKETS
C
      FIRST = 31
      LAST = 60
      DO 90 I=1,NBR
         DO 80 J=FIRST,LAST
              MONTH(I) = MONTH(I) + DEM(J)
80       CONTINUE
         FIRST = LAST + 1
```

```
      LAST = LAST + 30
90    CONTINUE
      IF (N .EQ. 0) RETURN 1
      RETURN
800   FORMAT(I1)
810   FORMAT(I1,46(4X,I4,F4.0,1X))
      END
```

```
      SUBROUTINE TSTATS(X,N,XMEAN,STDV,IER)
C*************************************************************************
C**                                                                     **
C**   TSTATS COMPUTES THE SAMPLE MEAN AND SAMPLE STANDARD               **
C**   DEVIATION OF AN INPUT DATA ARRAY WHEN CALLED FROM THE MAIN        **
C**   PROGRAM.  VARIOUS ERROR RETURN CODES SIGNIFY WHETHER THE          **
C**   SAMPLE SIZE IS TOO SMALL FOR PROCESSING OR THE STANDARD           **
C**   DEVIATION EQUALS ZERO.                                            **
C**                                                                     **
C**   DEFINITION OF VARIABLES:                                          **
C**                                                                     **
C**      AN    --  FLOATING POINT VALUE OF N                            **
C**      DEV   --  INTERMEDIATE VALUE OF STDV CALCULATION               **
C**      I     --  GENERAL DO INDEX                                     **
C**      IER   --  ERROR RETURN CODE EQUAL TO:                          **
C**                 0 -- NORMAL RETURN                                  **
C**                -1 -- INSUFFICIENT DATA (IE. 0 OR 1)                 **
C**                -3 -- STDV EQUALS 0                                  **
C**      N     --  NUMBER OF DATA ELEMENTS IN THE SAMPLE                **
C**      STDV  --  SAMPLE STANDARD DEVIATION OF DATA VECTOR X (N-1)     **
C**      SUM   --  DOUBLE PRECISION SUM OF OBSERVATIONS                 **
C**      X     --  INPUT DATA VECTOR OF LENGTH N                        **
C**      XMEAN --  SAMPLE MEAN OF DATA VECTOR X                         **
C**                                                                     **
C*************************************************************************
      INTEGER*4
     *   I,                IER,              .N
      REAL*4
     *   X(N),             XMEAN,            STDV,             AN,
     *   DEV
      REAL*8
     *   SUM
      AN = FLOAT(N)
      IER = 0
      IF(N .GE. 2) GO TO 10
         IER = -1
```

```
      GOTO 40
10    CONTINUE
C
C     COMPUTE MEAN AND MOMENT ESTIMATES
C     FIND THE MEAN
C
      SUM = 0.D0
      DO 20 I=1,N
         SUM = SUM + X(I)
20    CONTINUE
      XMEAN = SNGL(SUM) /AN
      SUM = 0.D0
      DO 30 I=1,N
         DEV = X(I) - XMEAN
         SUM = SUM + DEV * DEV
30    CONTINUE
      STDV = SQRT(SNGL(SUM) / (AN - 1.0))
      IF (STDV .EQ. 0.0) IER = -3
40    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE XINV(P,MU,STDV,FX,MODEL,NPTILE,X,IER)
C*****************************************************************************
C**                                                                         **
C**   XINV IS CALLED BY THE MAIN PROGRAM TO COMPUTE THE ABSCISSA            **
C**   VALUES FOR EACH MODEL BEING EVALUATED AT THE FIVE                     **
C**   PERCENTILES OF INTEREST, CURRENTLY THE 75TH, 80TH, 85TH,              **
C**   90TH AND 95TH PERCENTILES.  FOR THE FAMILY OF NORMAL                  **
C**   DISTRIBUTIONS, (NORMAL, LOGNORMAL AND BERNOULLI-LOGNORMAL)            **
C**   XINV CALLS THE IMSL SINGLE PRECISION SUBROUTINE MDNRIS TO             **
C**   FIND THE NORMALIZED ABSCISSA VALUE AND THEN CONVERTS THAT             **
C**   VALUE TO THE CORRECT INVERSE VALUE BY APPLYING THE CORRECT            **
C**   SAMPLE MEAN AND STANDARD DEVIATION CONVERSIONS.  FOR THE              **
C**   EXPONENTIAL, BERNOULLI-EXPONENTIAL AND BERNOULLI-LOGISTIC             **
C**   DISTRIBUTIONS, XINV COMPUTES THE ABSCISSA VALUE DIRECTLY FROM         **
C**   THE DATA SUPPLIED.  THE REMAINING DISTRIBUTIONS, POISSON,             **
C**   NEGATIVE BINOMIAL, LOGISTIC AND LA PLACE, XINV CALLS THE              **
C**   SUBROUTINES POISSN, NEGBIN, LOGIST AND LAPLCE RESPECTIVELY            **
C**   TO COMPUTE THE ABSCISSA VALUES.                                       **
C**   UPON RETURN TO THE MAIN PROGRAM, XINV HAS COMPUTED THE                **
C**   INVERSE FOR EACH SAMPLE AT EVERY PERCENTILE FOR EVERY MODEL.          **
C**                                                                         **
C**   COMPUTES THE INVERSE CUMULATIVE PROBABILITY FUNCTION FOR THE          **
C**   FOLLOWING DISTRIBUTIONS SELECTED BY THE VALUE OF THE PARAMETER        **
C**   MODEL:                                                                **
C**                                                                         **
C**      1.  POISSON                                                        **
C**      2.  NEGATIVE BINOMIAL                                              **
C**      3.  STANDARD NORMAL                                                **
C**      4.  EXPONENTIAL                                                    **
C**      5.  LOGNORMAL                                                      **
C**      6.  LOGISTIC                                                       **
C**      7.  LAPLACE                                                        **
C**      8.  BERNOULLI-EXPONENTIAL                                          **
C**      9.  BERNOULLI-LOGNORMAL                                            **
C**     10.  BERNOULLI-LOGISTIC                                             **
C**                                                                         **
```

```
C**   DEFINITION OF VARIABLES:                                           **
C**                                                                      **
C**      FX        -- PROBABILITY VALUE                                  **
C**      HX        -- COMPLEMENTARY PROBABILITY VALUE                    **
C**      IER       -- ERROR PARAMETER INDICATING                         **
C**                   0 NORMAL RETURN                                    **
C**                   NE 0 RETURN CODE FROM SUBROUTINE                   **
C**                   OTHER CODE RETURNED FROM ROUTINE "MDNRIS"          **
C**      MODEL     -- SELECTS DISTRIBUTION                               **
C**      MTYPE     -- DETERMINES WHICH MU & STDV EACH MODEL USES         **
C**      MU        -- SAMPLE MEAN OF EACH GROUP OF DATA                  **
C**      NPTILE    -- ARRAY OF PERCENTILES FOR TESTING                   **
C**      P         -- BERNOULLI PROBABILITY PARAMETER                    **
C**      PI        -- CONSTANT PI                                        **
C**      STDV      -- SAMPLE STANDARD DEVIATION OF EACH GROUP OF DATA    **
C**      X         -- ABSCISSA VALUE RESULT OF INVERSE FUNCTION           *
C**      Z         -- NORMALIZED RESULT OF INVERSE FUNCTIONS             **
C**                                                                      **
C**   OTHER SUBROUTINES REQUIRED BY XINV:                                **
C**                                                                      **
C**      LAPLCE  -- DISTRIBUTION, DENSITY OR INVERSE LAPLACE VALUES      **
C**      LOGIST  -- DISTRIBUTION, DENSITY OR INVERSE LOGISTIC VALUES     **
C**      MDNRIS  -- IMSL INVERSE NORMAL SUBROUTINE                       **
C**      NEGBIN  -- DISTRIBUTION, DENSITY OR INVERSE NEGATIVE            **
C**                 BINOMIAL VALUES                                      **
C**      POISSN  -- DISTRIBUTION, DENSITY OR INVERSE POISSON VALUES      **
C**                                                                      **
C**   THE MEAN AND STANDARD DEVIATION ARE ESTIMATED BY THE METHOD OF     **
C**   MOMENTS.  FOR DISTRIBUTIONS SUCH AS THE LAPLACE AND LOGISTIC,      **
C**   THESE ESTIMATORS ARE BIASED, BUT REMAIN THE CHOICE FOR USE IN      **
C**   THIS ANALYSIS DUE TO EASE OF COMPUTATION.                          **
C**                                                                      **
C************************************************************************
C
      REAL*4
     *   FACT,           FX(NPTILE),     MU(3),          P,
```

```
     *    PR,                 HX,                 PI/3.1415926/,  STDV(3),
     *    X(MODEL,NPTILE),                        Z
      INTEGER*4
     *    IER,        MODEL,      NPTILE,     MTYPE(10)/1,1,1,1,3,1,1,2,3,2/,
     *    J,          K
      IER = 0
      DO 130 J=1,MODEL
         DO 120 K=1,NPTILE
            HX = 1.0 - FX(K)
C
C           RETURN X = 0.0 FOR COMPOUND DISTRIBUTIONS IF P <= HX
C
            X(J,K) = 0.0
            IF ((J .GE. 8) .AND. (P .LE. HX)) GOTO 110
C
C           SELECT THE DISTRIBUTION
C
            GOTO (10,20,30,40,50,60,70,80,90,100),J
10          CONTINUE
C
C              INVERSE POISSON FUNCTION
C
               CALL POISSN(X(J,K),MU(MTYPE(J)),3,FX(K),IER)
               IF (IER .NE. 0) GOTO 130
            GOTO 110
20          CONTINUE
C
C              INVERSE NEGATIVE BINOMIAL FUNCTION
C
               CALL NEGBIN(X(J,K),MU(MTYPE(J)),
     *              STDV(MTYPE(J)),3,FX(K),IER)
               IF (IER .NE. 0) GOTO 130
            GOTO 110
30          CONTINUE
C
C              INVERSE STANDARD NORMAL FUNCTION
```

```
C
            CALL MDNRIS(FX(K),Z,IER)
            IF (IER .NE. 0) GOTO 130
            X(J,K) = MU(MTYPE(J)) + STDV(MTYPE(J)) * Z
        GOTO 110
40      CONTINUE
C
C           INVERSE EXPONENTIAL FUNCTION
C
            X(J,K) = -MU(MTYPE(J)) * LOG(HX)
        GOTO 110
50      CONTINUE
C
C           INVERSE LOG NORMAL FUNCTION
C
            CALL MDNRIS(FX(K),Z,IER)
            IF (IER .NE. 0) GOTO 140
            X(J,K) = EXP(MU(MTYPE(J)) + STDV(MTYPE(J)) * Z)
        GOTO 110
60      CONTINUE
C
C           INVERSE LOGISTIC FUNCTION
C
            CALL LOGIST(X(J,K),MU(MTYPE(J)),STDV(MTYPE(J)),
     *          3,FX(K),IER)
            IF (IER .NE. 0) GOTO 140
        GOTO 110
70      CONTINUE
C
C           INVERSE LAPLACE FUNCTION
C
            CALL LAPLCE(X(J,K),MU(MTYPE(J)),STDV(MTYPE(J)),
     *          3,FX(K),IER)
            IF (IER .NE. 0) GOTO 140
        GOTO 110
80      CONTINUE
```

```
C
C                 INVERSE MIXED BERNOULLI-EXPONENTIAL FUNCTION
C
                  X(J,K) = MU(MTYPE(J)) * LOG(P / HX)
               GOTO 110
90             CONTINUE
C
C                 INVERSE BERNOULLI-LOGNORMAL FUNCTION
C
                  PR = (P - HX) / P
                  CALL MDNRIS(PR,Z,IER)
                  IF (IER .NE. 0) GOTO 140
                  X(J,K) = EXP(MU(MTYPE(J)) + STDV(MTYPE(J)) * Z)
               GOTO 110
100            CONTINUE
C
C                 INVERSE BERNOULLI-LOGISTIC FUNCTION
C
                  FACT = SQRT(3.0) * STDV(MTYPE(J)) / PI
                  Z = HX * (1.0 + TANH(MU(MTYPE(J)) / (2.0 * FACT)))
                  X(J,K) = MU(MTYPE(J)) + FACT * LOG(2.0 * P / Z - 1.0)
110            CONTINUE
120         CONTINUE
130      CONTINUE
140      CONTINUE
         RETURN
         END
```

```
      SUBROUTINE POISSN (X,MU,TYPE,FX,IER)
C*********************************************************************
C**                                                                 **
C**   POISSN IS A GENERAL PURPOSE POISSON DISTRIBUTION FUNCTION     **
C**   THAT RETURNS EITHER THE DENSITY OR CUMULATIVE DISTRIBUTION    **
C**   VALUE GIVEN THE ABSCISSA VALUE OR ALTERNATIVELY, RETURNS THE  **
C**   ABSCISSA VALUE GIVEN THE PROBABILITY.                         **
C**   COMPUTES POISSON DENSITY AND DISTRIBUTION FUNCTIONS BY USE OF **
C**   RECURSION EQUATIONS.                                          **
C**                                                                 **
C**      X - INPUT PARAMETER REPRESENTING ABCISSA VALUES OF THE     **
C**          POISSON DISTRIBUTION.  FOR ITYPE = 3, X IS AN OUTPUT   **
C**          PARAMETER.                                             **
C**                                                                 **
C**      MU - MEAN OF THE DESIRED POISSON DISTRIBUTION.             **
C**                                                                 **
C**      TYPE = 1  RETURNS PDF IN FX GIVEN X                        **
C**             2  RETURNS CDF IN FX GIVEN X                        **
C**             3  RETURNS INVERSE CDF VALUES IN X GIVEN FX AS CDF  **
C**                                                                 **
C**      FX- OUTPUT PARAMETER OF PROBABILITY (X<=X).  FOR ITYPE = 3,**
C**          FX IS AN INPUT PARAMETER.                              **
C**                                                                 **
C**      IER - ERROR PARAMETER RETURN CODE EQUAL TO:                **
C**           0 - NORMAL RETURN                                     **
C**          -1 - INVALID SELECTION OF TYPE                         **
C**          -3 - ALGORITHM DID NOT CONVERGE WITHIN 1000 ITERATIONS.**
C**                                                                 **
C*********************************************************************
      INTEGER*4
     *   I,               IER,              TYPE
      REAL*4
     *   CDF,             PDF,              FX,              MU,
     *   A,               X,                ZL/1.0E-20/
      PDF = 0.0
      IF (MU .LT. 170.0) PDF = EXP(-MU)
```

```
      CDF = PDF
      A = 0.0
      DO 50 I=1,1000
         IF (TYPE .LT. 1) GOTO 60
         IF (TYPE .GT. 3) GOTO 60
         GOTO (10,20,30),TYPE
10       CONTINUE
            IF (A .LT. X) GOTO 40
               FX = PDF
               GOTO 99
20       CONTINUE
            IF (A .LT. X) GOTO 40
               FX = CDF
               GOTO 99
30       CONTINUE
            IF (CDF .LT. FX) GOTO 40
               X = A
               GOTO 99
40       CONTINUE
         A = A + 1.0
         IF (PDF .GT. ZL) PDF = MU * PDF / A
         IF (PDF .LE. ZL) PDF = 0.0
         CDF = CDF + PDF
50    CONTINUE
      IER = -3
      GOTO 99
60    CONTINUE
      IER = -1
99    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE NEGBIN  (X,MU,STDV,TYPE,FX,IER)
C*********************************************************************
C**                                                                 **
C**   NEGBIN IS A GENERAL PURPOSE NEGATIVE BINOMIAL DISTRIBUTION    **
C**   FUNCTION THAT RETURNS EITHER THE DENSITY OR CUMULATIVE        **
C**   DISTRIBUTION VALUE GIVEN THE ABSCISSA VALUE, OR ALTERNATIVELY **
C**   RETURNS THE ABSCISSA VALUE GIVEN THE PROBABILITY.             **
C**   COMPUTES NEGATIVE BINOMIAL DENSITY AND DISTRIBUTION FUNCTIONS **
C**   BY USE OF RECURSION EQUATIONS.                                **
C**                                                                 **
C**      X - INPUT PARAMETER REPRESENTING ABSCISSA VALUES OF THE    **
C**          NEGETIVE BINOMIAL DISTRIBUTION.  FOR TYPE = 3, X IS    **
C**          AN OUTPUT PARAMETER.                                   **
C**                                                                 **
C**      MU - MEAN OF THE NEGATIVE BINOMIAL DISTRIBUTION.           **
C**                                                                 **
C**      STDV - STANDARD DEVIATION OF THE NEGATIVE BINOMIAL         **
C**             DISTRIBUTION.                                       **
C**                                                                 **
C**      TYPE = 1  RETURNS PDF IN FX GIVEN X                        **
C**             2  RETURNS CDF IN FX GIVEN X                        **
C**             3  RETURNS INVERSE CDF VALUES IN X GIVEN FX AS CDF  **
C**                                                                 **
C**      FX- OUTPUT PARAMETER OF PROBABILITY (X<=X).  FOR TYPE = 3, **
C**          FX IS AN INPUT PARAMETER.                              **
C**                                                                 **
C**      IER - ERROR PARAMETER RETURN CODE EQUAL TO:                **
C**            0 - NORMAL RETURN                                    **
C**           -1 - INVALID SELECTION OF TYPE                        **
C**           -3 - ALGORITHM DID NOT CONVERGE WITHIN 1000 ITERATIONS.**
C**                                                                 **
C*********************************************************************
      INTEGER*4
     *   I,                IER,              TYPE
      REAL*4
     *   CDF,              PDF,              FX,               MU,
```

```
     *   STDV,            VAR,            A,              X,
     *   RHO,             ONE/1.0/,       K,              ZL/1.0E-20/
      VAR = STDV**2
      RHO = MU / VAR
      K = MU**2 / (VAR - MU)
      PDF = RHO**K
      CDF = PDF
      A = 0.0
      DO 50 I=1,1000
         IF (TYPE .LT. 1) GOTO 60
         IF (TYPE .GT. 3) GOTO 60
         GOTO (10,20,30),TYPE
10            CONTINUE
                 IF (A .LT. X) GOTO 40
                    FX = PDF
                    GOTO 99
20            CONTINUE
                 IF (A .LT. X) GOTO 40
                    FX = CDF
                    GOTO 99
30            CONTINUE
                 IF (CDF .LT. FX) GOTO 40
                    X = A
                    GOTO 99
40       CONTINUE
         A = A + ONE
         IF (PDF .GT. ZL) PDF = (A + K - ONE) * (ONE - RHO) * PDF / A
         IF (PDF .LE. ZL) PDF = 0.0
         CDF = CDF + PDF
50    CONTINUE
      IER = -3
      GOTO 99
```

```
60    CONTINUE
      IER = -1
99    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LOGIST (X,MU,STDV,TYPE,FX,IER)
C***********************************************************************
C**                                                                  **
C**   LOGIST IS A GENERAL PURPOSE LOGISTIC DISTRIBUTION FUNCTION     **
C**   THAT RETURNS EITHER THE DENSITY OR CUMULATIVE DISTRIBUTION     **
C**   VALUE GIVEN THE ABSCISSA VALUE OR ALTERNATIVELY, RETURNS THE   **
C**   ABSCISSA VALUE GIVEN THE PROBABILITY.                          **
C**                                                                  **
C**      X - INPUT PARAMETER REPRESENTING THE ABSCISSA VALUE OF      **
C**          THE LOGISTIC DISTRIBUTION.  FOR TYPE = 3, X IS AN       **
C**          OUTPUT PARAMETER.                                       **
C**                                                                  **
C**      MU- MEAN OF THE LOGISTIC DISTRIBUTION OR METHOD OF MOMENTS  **
C**          OR MLE ESTIMATE.                                        **
C**                                                                  **
C**      STDV- STANDARD DEVIATION OF THE LOGISTIC DISTRIBUTION OR    **
C**          METHOD OF MOMENTS OR MLE ESTIMATE.                      **
C**                                                                  **
C**      TYPE = 1  RETURNS PDF IN FX GIVEN X                         **
C**             2  RETURNS CDF IN FX GIVEN X                         **
C**             3  RETURNS INVERSE CDF VALUES IN X GIVEN FX AS CDF   **
C**                                                                  **
C**      FX- OUTPUT PARAMETER OF PROBABILITY (X<=X).  FOR TYPE = 3,  **
C**          FX IS AN INPUT PARAMETER.                               **
C**                                                                  **
C**      IER - ERROR PARAMETER RETURN CODE EQUAL TO:                 **
C**           0 - NORMAL RETURN                                      **
C**          -1 - INVALID SELECTION OF TYPE                          **
C**                                                                  **
C***********************************************************************
      INTEGER
     *   IER,           TYPE
      REAL*4
     *   ARG,           FX,            HALF/0.5/,     ONE/1.0/,
     *   PI/3.1415926/, SECHX,         SQRPI,         X,
     *   MU,            STDV
```

```
C
C     INITIALIZE CONSTANTS
C
      SQRT3 = SQRT(3.0)
      ARG = HALF * PI * (X - MU) / (SQRT3 * STDV)
      IF (TYPE .LT. 1) GOTO 40
      IF (TYPE .GT. 3) GOTO 40
      GOTO (10,20,30), TYPE
10       CONTINUE
C
C           COMPUTE DISTRIBUTION PDF
C
            SECHX = ONE / COSH(ARG)
            FX = PI * SECHX**2 / (4.0 * SQRT3 * STDV)
            GOTO 50
20       CONTINUE
C
C           COMPUTE DISTRIBUTION CDF
C
            FX = HALF * (ONE + TANH(ARG))
            GOTO 50
30       CONTINUE
C
C           COMPUTE X VALUES FROM PROBABILITY INTEGRAL TRANSFORM
C
            X = MU + STDV * SQRT3 * LOG(FX / (ONE - FX)) / PI
            GOTO 50
40       CONTINUE
            IER = -1
50    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LAPLCE  (X,MU,STDV,TYPE,FX,IER)
C*********************************************************************
C**                                                                 **
C**   LAPLCE IS A GENERAL PURPOSE LAPLACE DISTRIBUTION FUNCTION     **
C**   THAT RETURNS EITHER THE DENSITY OR CUMULATIVE DISTRIBUTION    **
C**   VALUE GIVEN THE ABSCISSA VALUE OR ALTERNATIVELY, RETURNS THE  **
C**   ABSCISSA VALUE GIVEN THE PROBABILITY.                         **
C**                                                                 **
C**      X - INPUT PARAMETER REPRESENTING THE ABSCISSA VALUE OF     **
C**          THE LAPLACE DISTRIBUTION.  FOR TYPE = 3, X IS AN       **
C**          OUTPUT PARAMETER.                                      **
C**                                                                 **
C**      MU- MEAN OF THE LAPLACE DISTRIBUTION OR METHOD OF MOMENTS  **
C**          OR MLE ESTIMATE.                                       **
C**                                                                 **
C**      STDV- STANDARD DEVIATION OF THE LAPLACE DISTRIBUTION OR    **
C**          METHOD OF MOMENTS OR MLE ESTIMATE.                     **
C**                                                                 **
C**      TYPE = 1  RETURNS PDF IN FX GIVEN X                        **
C**             2  RETURNS CDF IN FX GIVEN X                        **
C**             3  RETURNS INVERSE CDF VALUES IN X GIVEN FX AS CDF  **
C**                                                                 **
C**      FX- OUTPUT PARAMETER OF PROBABILITY (X<=X).  FOR TYPE = 3, **
C**          FX IS AN INPUT PARAMETER.                              **
C**                                                                 **
C**      IER - ERROR PARAMETER RETURN CODE EQUAL TO:                **
C**           0 - NORMAL RETURN                                     **
C**          -1 - INVALID SELECTION OF TYPE                         **
C**                                                                 **
C*********************************************************************
      INTEGER
     *   IER,           TYPE
      REAL*4
     *   ARG,           FX,             ONE/1.0/,       SQR2/1.4142135/,
     *   TWO/2.0/,      X,              ZERO/0.0/,      MU,
     *   STDV
```

```
C
C     INITIALIZE CONSTANTS
C
      ARG = SQR2 * ABS(X - MU) / STDV
      IF (TYPE .LT. 1) GOTO 40
      IF (TYPE .GT. 3) GOTO 40
      GOTO (10,20,30), TYPE
10       CONTINUE
C
C           COMPUTE DISTRIBUTION PDF
C
            FX = EXP(-ARG) / (SQR2 * STDV)
            GOTO 50
20       CONTINUE
C
C           COMPUTE DISTRIBUTION CDF
C
            FX = EXP(-ARG) / TWO
            IF (X .GE. ZERO) FX = ONE - FX
            GOTO 50
30       CONTINUE
C
C           COMPUTE X VALUES FROM PROBABILITY INTEGRAL TRANSFORM
C
            IF (FX .LT. 0.5) X = MU + STDV * LOG(TWO * FX) / SQR2
            IF (FX .GE. 0.5) X = MU - STDV * LOG(TWO - TWO * FX)/SQR2
            GOTO 50
40       CONTINUE
            IER = -1
50    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SAMPLE(ISEED,OBS,NBR,I,MODEL,NPTILE,X,FX,PHAT,MSE)
C*****************************************************************
C**                                                             **
C**   SAMPLE IS THE HEART OF THE ANALYSIS AND IS CALLED REPEATEDLY **
C**   BY THE MAIN PROGRAM TO GENERATE SUCCESSIVE REPETITIONS OF   **
C**   THE PSEUDO SAMPLES FOR THE PURPOSE OF AVERAGING THE RESULTS. **
C**   THE RANDOM INTEGERS FROM ONE TO FORTY-EIGHT ARE OBTAINED    **
C**   EXPANDING FORTY-EIGHT ZERO TO ONE RANDOM NUMBERS BY         **
C**   MULTIPLYING BY FORTY-EIGHT, ADDING ONE AND TRUNCATING TO    **
C**   INTEGERS.  THE PSEUDO SAMPLE IS THEN CREATED FROM THE EDITED **
C**   DEMAND OBSERVATIONS AS DESCRIBED IN SECTION IV.B.           **
C**   EACH SAMPLE IS ANALYZED BY SUCCESSIVELY CHECKING EACH       **
C**   OBSERVATION AGAINST THE DESIRED ABSCISSA VALUE COMPUTED     **
C**   EARLIER BY XINV AND RECORDING A SUCCESS IF THE SAMPLE       **
C**   OBSERVATION IS LESS THAN OR EQUAL TO THE CURRENT ABSCISSA   **
C**   VALUE.  SUCCESSES ARE REFLECT BY ADDING ONE TO THE VARIABLE **
C**   SUCCES WHILE FAILURES ADD ZERO TO THE VARIABLE.  TO HANDLE  **
C**   THE CASE OF INTEGER ABSCISSA VALUE, FOR THE POISSON AND     **
C**   NEGATIVE BINOMIAL DISTRIBUTIONS AND ALSO FOR THE COMPOUND   **
C**   DISTRIBUTIONS WHERE THE ABSCISSA VALUE IS ZERO, SUCCESSES   **
C**   WERE RECORDED ONLY UNTIL THE NUMBER OF SUCCESSES WERE LESS  **
C**   THAN THE EXPECTED NUMBER OF SUCCESSES (COMPUTED AS THE      **
C**   PERCENTILE TIMES THE NUMBER OF OBSERVATIONS--FORTY EIGHT).  **
C**   AFTER THE ENTIRE PSEUDO SAMPLE HAS BEEN CHECKED, THE RESAMPLING **
C**   PROCEDURE ESTIMATE OF P IS COMPUTED AND ADDED TO THE        **
C**   ACCUMULATOR VARIABLE PHAT FOR LATER OUTPUT.  LIKEWISE, THE  **
C**   MEAN SQUARED ERROR IS COMPUTED AND ACCUMULATED IN THE       **
C**   VARIABLE MSE FOR LATER OUTPUT                               **
C**                                                             **
C**   VARIABLES USED ARE AS FOLLOWS:                              **
C**                                                             **
C**      A         -- DUMMY ARRAY FOR RANDOM NUMBERS FOR SAMPLERAP **
C**      AN        -- FLOAT OF NBR                                **
C**      BPROB     -- RESAMPLING PROBABILITY OF DEMAND LT EXPECTED **
C**      EXPS      -- EXPECTED NUMBER OF BERNOULLI SUCCESSES      **
C**      FX        -- PRABABILITIES OF INTEREST                   **
```

```
C**      I           -- DEMAND CATEGORY                                      **
C**      IPRT        -- CONTROL PARAMETER FOR OUTPUT OF DIAGNOSTIC INFO      **
C**      IPRINT      -- DSRN FOR PRINTER DISK                                **
C**      ISEED       -- SEED FOR RANDOM NUMBER GENERATOR LRND                **
C**      J           -- DO INDEX SEQUENCING THROUGH MODELS                   **
C**      K           -- DO INDEX SEQUENCING THROUGH PERCENTILES              **
C**      L           -- DO INDEX SEQUENCING THROUGH SAMPLERAP SAMPLES        **
C**      IND         -- INDEX TO SELECT RANDOM OBSERVATION FROM OBS          **
C**      MODEL       -- MUNBER OF MODELS TO TEST                             **
C**      MSE         -- MEAN SQUARED ERROR OF PERCENTILES                    **
C**      NBR         -- NUMBER OF OBSERVATIONS IN SAMPLE                     **
C**      NPTILE      -- NUMBER OF PERCENTILES TO TEST                        **
C**      OBS         -- ARRAY OF DEMAND OBSERVATIONS AND DERIVATIVES         **
C**      PHAT        -- CUMULATIVE PERCENTILE ESTIMATES                      **
C**      SAMP        -- PSEUDO SAMPLE OF DEMAND OBSERVATIONS                 **
C**      SUCCES      -- NUMBER OF BERNOULLI SUCCESSES EACH TRIAL             **
C**      X           -- EXPECTED DEMAND OBSERVATION FOR A GIVEN              **
C**                     PERCENTILE AND MODEL                                 **
C**                                                                          **
C*****************************************************************************
      INTEGER*4
     *   ISEED,          NBR,            MODEL,          NPTILE,
     *   I,              J,              K,              L,
     *   SUCCES,         IND,            IPRT,           IPRINT
      REAL*4
     *   AN,                          BPROB,
     *   SAMP(48),                    PHAT(3,MODEL,NPTILE),
     *   MSE(3,MODEL,NPTILE),         X(MODEL,NPTILE),
     *   OBS(NBR,3),                  FX(NPTILE),
     *   A(48),                       EXPS(10)
      COMMON /FILES/
     *   IPRT,           ICONIN,         ICNOUT,         IPRINT,
     *   IDSK(30)
      AN = FLOAT(NBR)
      DO 10 K=1,NPTILE
         EXPS(K) = FX(K) * AN
```

```
10    CONTINUE
C
C     GENERATE PSEUDO SAMPLES
C
      CALL LRND(ISEED,A,NBR,2,0)
      DO 20 L=1,NBR
           IND = IFIX(A(L) * AN) + 1
           SAMP(L) = OBS(IND,1)
20    CONTINUE
      IF (IPRT .GE. 5) WRITE(IPRINT,600) (SAMP(L),L=1,NBR)
C
C     COMPUTE SAMPLE PERCENTILES FOR EACH PSEUDO SAMPLE
C
      DO 90 J=1,MODEL
         IF (IPRT .GE. 5) WRITE(IPRINT,610) J
         DO 80 K=1,NPTILE
              SUCCES = 0
              DO 70 L=1,NBR
                   IF (.NOT.(SAMP(L) .LE. X(J,K))) GOTO 60
                        IF (AMOD(X(J,K),1.0) .EQ. 0.0) GOTO 30
C
C                            FOR NON-INTEGER VALUES OF EXPECTED DEMAND,
C                            ACCUMULATE BERNOULLI SUCCESSES WHENEVER
C                            ACTUAL DEMAND, REPRESENTED BY THE PSEUDO
C                            SAMPLE, IS LESS THAN OR EQUAL TO EXPECTED
C                            DEMAND.
C
                             SUCCES = SUCCES + 1
                        GOTO 50
30                      CONTINUE
C
C                            IN THE CASE OF INTEGER VALUES OF EXPECTED
C                            DEMAND, SUCCESSES ARE ACCUMULATED UNTIL
C                            THE EXPECTED NUMBER OF SUCCESSES IS
C                            REACHED.
C
```

```
                        IF (FLOAT(SUCCES) .GE. EXPS(K)) GOTO 40
                           SUCCES = SUCCES + 1
40                      CONTINUE
50                   CONTINUE
60                CONTINUE
70             CONTINUE
               BPROB = FLOAT(SUCCES) / AN
               IF (IPRT .GE. 5) WRITE(IPRINT,620) X(J,K),BPROB,FX(K)
C
C              ACCUMULATE SQUARED ERROR AND ESTIMATED PERCENTILE
C              FOR EACH DISTRIBUTION AT EACH THEORETICAL PERCENTILE
C
               PHAT(I,J,K) = PHAT(I,J,K) + BPROB
               MSE(I,J,K) = MSE(I,J,K) + (BPROB - FX(K))**2
               IF(IPRT .GE. 5) WRITE(IPRINT,630) PHAT(I,J,K),MSE(I,J,K)
80          CONTINUE
90       CONTINUE
         RETURN
600      FORMAT('0PSEUDO SAMPLE ',3(T20,20F5.0,/))
610      FORMAT('0**** MODEL ****    ',I1)
620      FORMAT('0PROBABILITY PSEUDO SAMPLE .LE. ',F12.6,' IS',F12.6,
     *      ' AT THE ',F4.2,' PERCENTILE')
630      FORMAT('0CUMULATIVE:  PROBABILITY ',F5.1,'  SQUARED ERROR; 'F12.6)
         END
```

```
      SUBROUTINE OUTPUT(MODEL,NPTILE,NREPS,FX,IC,PHAT,MSE,CAT)
C******************************************************************
C**                                                              **
C**   OUTPUT IS THE FINAL SUBROUTINE CALLED BY THE MAIN PROGRAM  **
C**   AND IS USED TO GENERATE THE OUTPUT REPORT.  ONLY MINOR     **
C**   COMPUTATIONS ARE PERFORMED BY OUTPUT, THOSE BEING COMPUTING **
C**   THE AVERAGE P VALUE AND ITS STANDARD DEVIATION, AVERAGING  **
C**   THE MEAN SQUARED ERROR AND TOTALING THE AVERAGE MEAN SQUARED **
C**   ERROR FOR EACH DEMAND CLASS AND MODEL.                     **
C**                                                              **
C**   DEFINITION OF VARIABLES:                                   **
C**                                                              **
C**      CAT      -- BREAKPOINTS OF DEMAND CATEGORIES FOR LOW,   **
C**                  MEDIUM AND HIGH DEMANDS                     **
C**      DISTR    -- CHARACTER NAME OF EACH MODEL TESTED         **
C**      FX       -- DISTRIBUTION PERCENTILES TO TEST            **
C**      I        -- DEMAND CATEGORY FOR SAMPLE UNDER TEST       **
C**      IC       -- COUNT OF SAMPLES IN EACH DEMAND CATEGORY    **
C**      IPRINT   -- DSRN OF PRINTER DISK                        **
C**      J        -- DO INDEX SEQUENCING THROUGH MODELS          **
C**      K        -- DO INDEX SEQUENCING THROUGH PERCENTILES     **
C**      MODEL    -- NUMBER OF MODELS TO STUDY                   **
C**      MSE      -- MEAN SQUARED ERROR OF PREDICTED VS. ACTUAL  **
C**                  PERCENTILE                                  **
C**      MU       -- BERNOULLI MEAN OF PSEUDO SAMPLE REPETITIONS **
C**                  FOR EACH PERCENTILE AND MODEL TESTED        **
C**      NPTILE   -- NUMBER OF SAMPLE PERCENTILES TO TEST        **
C**      NREPS    -- NUMBER OF PSEUDO SAMPLE REPETITIONS         **
C**      PHAT     -- CUMULATIVE PERCENTILE ESTIMATES FOR EACH    **
C**                  DEMAND CATEGORY, MODEL AND PERCENTILE       **
C**      STDV     -- BERNOULLI STANDARD DEVIATION OF PSEUDO SAMPLE **
C**                  REPETITIONS FOR EACH PERCENTILE AND         **
C**                  MODEL TESTED                                **
C**      TMSE     -- TOTAL MEAN SQUARE ERROR SUMMED OVER PERCENTILES **
C**                  FOR EACH DEMAND CATEGORY AND MODEL          **
C**      TRIALS   -- TOTAL NUMBER OF REPETITIONS.  USED TO NORMALIZE **
```

```
C**                     FINAL FIGURES. - COMPUTED AS COUNT OF LINE ITEMS **
C**                     PER DEMAND CATEGORY TIMES NREPS                  **
C**                                                                      **
C*************************************************************************
      INTEGER*4
     *   I,            J,            K,            IC(3),
     *   MODEL,        NREPS,        NPTILE,       IPRINT
      REAL*4
     *   FX(NPTILE),                 CAT(4),
     *   MSE(3,MODEL,NPTILE),        PHAT(3,MODEL,NPTILE),
     *   TMSE,                       MU(10),
     *   STDV(10),                   TRIALS
      REAL*8
     *   RANGE(3)/'LOW','MEDIUM','HIGH'/
      COMPLEX*32
     *   DISTR(10)/'POISSON',                'NEGATIVE BINOMIAL',
     *             'NORMAL',                 'EXPONENTIAL',
     *             'LOGNORMAL',              'LA PLACE',
     *             'LOGISTIC',               'BERNOULLI-EXPONENTIAL',
     *             'BERNOULLI-LOGNORMAL',    'BERNOULLI-LOGISTIC'/
      COMMON /FILES/
     *   IPRT,         ICONIN,       ICNOUT,       IPRINT,
     *   IDSK(30)
      CAT(2) = CAT(2) * 12.0
      CAT(3) = CAT(3) * 12.0
      DO 30 I=1,3
         TRIALS = FLOAT(IC(I) * NREPS)
         IF (TRIALS .EQ. 0.0) GOTO 30
         DO 20 J=1,MODEL
            IF ((J .EQ. 1) .OR. (J .EQ. 4) .OR. (J .EQ. 8))
     *      WRITE(IPRINT,600) NREPS,IC(I),RANGE(I),CAT(I),CAT(I+1)
            WRITE(IPRINT,610) DISTR(J),(FX(K),K=1,NPTILE)
            TMSE = 0.0
            DO 10 K=1,NPTILE
               MU(K) = PHAT(I,J,K) / TRIALS
               STDV(K) = SQRT(MU(K) * (1.0 - MU(K))/ TRIALS)
```

```
                  MSE(I,J,K) = MSE(I,J,K) / TRIALS
                  TMSE = TMSE + MSE(I,J,K)
10             CONTINUE
               WRITE(IPRINT,620) (MU(K),K=1,NPTILE)
               WRITE(IPRINT,630) (STDV(K),K=1,NPTILE)
               TMSE = TMSE * 1.0E04
               WRITE(IPRINT,640) (MSE(I,J,K),K=1,NPTILE),TMSE
20          CONTINUE
30       CONTINUE
         RETURN
600      FORMAT(6X,I2,' REPETITIONS FOR EACH OF ',I4,' PSEUDO SAMPLES',/,
     *      6X,A7,'DEMAND RANGE:',F5.1,' < D < ',F7.2,' PER YEAR')
610      FORMAT(/,' DISTRIBUTION:  ',2A16,/,
     *      ' ESTIMATES',22X,'PERCENTILES (P)',/,12X,5F10.2)
620      FORMAT(' MEAN        ',5F10.6)
630      FORMAT(' STD DEV     ',5F10.6)
640      FORMAT(' MSE         ',5F10.6,/,' TOTAL MSE: ',F8.2,'E-04')
         END
```

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